ANIQ INTEGRAL YORDAMIDA TEKIS SHAKLNING YUZINI HISOBLASH

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Anotatsiya: *Ushbu maqolada aniq integral yordamida tekis shakllarning (egri chiziqlar bilan chegaralangan sohalarning) yuzalarini hisoblash usullari ko‘rib chiqiladi. Har bir usul tushuntirilgan va ularning amaliy qo‘llanilishi misollar bilan ko‘rsatilgan. Maqolada integral hisobning geometriyadagi ahamiyati, qo‘llanilishi qisqacha bayon etilgan.*

Kalit so‘zlar: *Aniq integral, tekis shakl yuzasi, egri chiziq, geometrik tadbiqlar.*

Ключевые слова: *Определенный интеграл, плоская поверхность, кривая, геометрические приложения.*

Keywords: *Definite integral, plane surface, curve, geometric applications.*

KIRISH

Aniq integral-matematik analizning asosiy tushunchalaridan biri bo‘lib, u yordamida turli geometrik masalalar, jumladan, figuralar yuzalarini hisoblash, hajmlarni topish va egri chiziqlar uzunliklarini aniqlash mumkin. Ushbu maqolada aniq integralning geometriyadagi asosiy qo‘llanilish misollarini ko‘rib chiqamiz.

Aytaylik, $f(x)$ funksiya $[a,b]$ segmentda berilgan va shu segmentda uzluksiz bo‘lsin. U holda $f(x)$ boshlang‘ich funksiya

$$F(x)=\int_a^x f(t)dt$$

ga ega bo‘ladi.

Ravshanki, $\Phi(x)$ funksiya $f(x)$ ning ixtiyoriy boshlang‘ich funksiyasi bo‘lsa, u holda

$$\Phi(x)=F(x)+C \qquad (C=const)$$

bo‘ladi.

Bu tenglikda, avval $x=a$ deb

$$\Phi(a)=C,$$

so‘ngra $x=b$ deb

$$\Phi(b)=\int_a^b f(x)dx+C$$

bo‘lishini topamiz. Demak,

$$\int_a^b f(x)dx=\Phi(b)-\Phi(a) \qquad (1)$$

(1) formula Nyuton-Leybnis formulasi deyiladi.

1. Tekis shaklning yuzi tushunchasi. Ma'lumki, (x, y) juftlik, $(x \in R, y \in R)$, tekislikda nuqtani ifodalaydi.

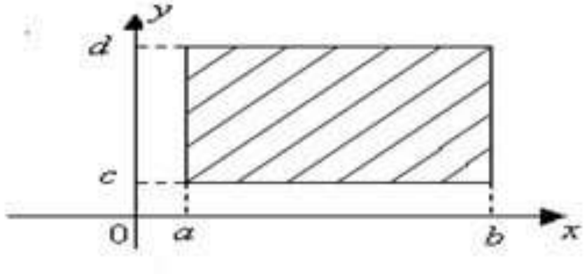
Koordinatalari ushbu

$$a \le x \le b, \quad c \le y \le d \qquad (a \in R, b \in R, c \in R, d \in R)$$

tengsizliklarni qanoatlantiruvchi tekislik nuqtalaridan hosil bo'lgan D_0 to'plam :

$$D_0 = \{(x, y); x \in [a,b], y \in [c,d]\}$$

to'g'ri to'rtburchak deyiladi.



1-chizma

Bu to'g'ri to'rtburchakning tomonlari (chegaralari) mos ravishda koordinatalar o'qiga parallel bo'ladi.

D_0 to'g'ri to'rtburchakning yuzi deb (uning chegarasining, ya'ni

$$x = a \quad , \quad x = b \qquad\qquad (c \le y \le d),$$

$$y = c \quad , \quad y = d \qquad\qquad (a \le x \le b)$$

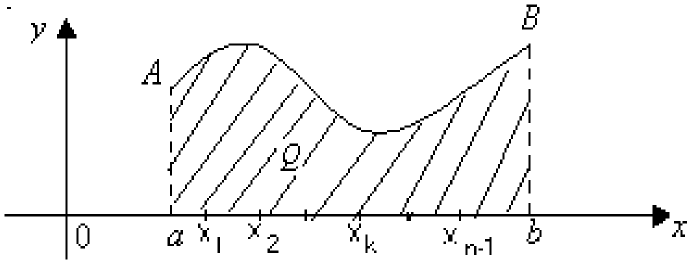
to'g'ri chiziq kesmalarining D_0 ga tegishli bo'lishi yoki tegishli bo'lmasligidan qat'iy nazar) ushbu

$$\mu(D_0) = (b - a) \cdot (d - c)$$

miqdorga aytiladi.

2. Egri chizikli trapesiyaning yuzini hisoblash. Faraz qilaylik, $f(x) \in C[a,b]$ bo'lib, $\forall x \in [a,b]$ da $f(x) \ge 0$ bo'lsin.

Yuqoridan $f(x)$ funksiya grafigi, yon tomonlardan $x = a$, $x = b$ vertikal chiziqlar hamda pastdan absissa o'qi bilan chegaralangan Q shaklni qaraylik.



2-chizma

Odatda, bu shakl egri chiziqli trapesiya deyiladi. Egri chiziqli trapesiyaning yuzi

$$\mu(Q)=\int_a^b f(x)dx \qquad (2)$$

ga teng bo‘ladi.

1-misol. Tekislikda ushbu

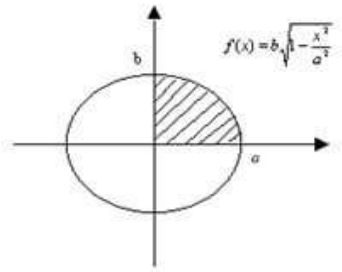
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

ellips bilan chegaralangan Q shaklning yuzi topilsin.

Ellips bilan chegaralangan Q shaklning yuzi OX va OY koordinata o‘qlari hamda

$$f(x)=b\cdot\sqrt{1-\frac{x^2}{a^2}} \qquad , \qquad 0\le x\le a$$

chiziqlar bilan chegaralangan egri chiziqli trapesiya yuzining 4 tasiga teng bo‘ladi.



3-chizma

Unda (2) formuladan foydalanib topamiz:

$$\mu(Q)=4\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx=\frac{4b}{a}\int_0^a\sqrt{a^2-x^2}\,dx=$$

$$=\left|\begin{array}{ll} x=a\sin t\ , & \\ & 0\le t\le\frac{\pi}{2} \\ dx=a\cos t dt\ , & \end{array}\right|=$$

$$=\frac{4b}{a}\cdot a^2\int_0^{\frac{\pi}{2}}\cos^2 t dt=4ab\cdot\frac{\pi}{4}=ab\pi.$$

Aytaylik, $f_1(x)\in C[a,b]$, $f_2(x)\in C[a,b]$ bo‘lib, $\forall x\in[a,b]$ da

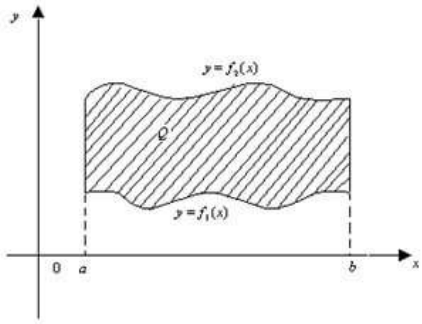
$$0\le f_1(x)\le f_2(x)$$

bo‘lsin.

Tekislikdagi Q shakl quyidagi

$$y=f_1(x) \quad , \quad y=f_2(x) \quad , \quad x=a \quad , \quad x=b$$

chiziqlar bilan chegaralangan shaklni ifodalasin.



4-chizma

Bu shaklning yuzi

$$\mu(Q)=\int_a^b f_2(x)dx-\int_a^b f_1(x)dx=\int_a^b\left[f_2(x)-f_1(x)\right]dx \qquad (3)$$

bo‘ladi.

2-misol. Tekislikda ushbu

$$y=4-x^2 \quad , \qquad y=x^2-2x$$

chiziqlar (parabolalar) bilan chegaralangan Q shaklning yuzi topilsin.

Parabolalarning tenglamalari

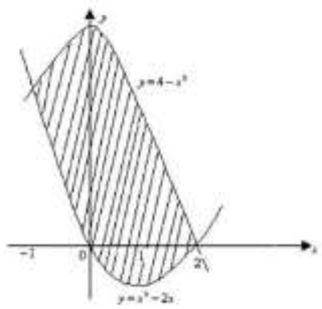
$$y=4-x^2 \quad ,$$

$$y=x^2-2x$$

ni birgalikda yechib, ularning kesishish nuqtalarini topamiz:

$$4-x^2=x^2-2x \ ,$$

$$x_1=-1\,,\ x_2=2\,;\ y_1=3\,,\ y_2=0:\ \ A(-1;3)\,,\ B(2;0).$$



5-chizma

Bu shaklning yuzini (3) formuladan foydalanib hisoblaymiz:

$$\mu(Q)=\int_{-1}^{2}\left[(4-x^2)-(x^2-2x)\right]dx=\int_{-1}^{2}\left(4+2x-2x^2\right)dx=(4x+x^2-\frac{2}{3}x^3)\Big|_{-1}^{2}=9.$$

Xulosa

Aniq integral geometriyada keng qo‘llaniladi: yuzalar, hajmlar, egri uzunliklarni hisoblashdan tortib, murakkab figuralarning xossalarni o‘rganishgacha. Uning yordami bilan aniq va umumiy formulalar ishlab chiqish mumkin, bu esa matematik modellashtirishda muhim rol o‘ynaydi.

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