



UCHBURCHAKLARNI YECHISHDA CHEVA TEOREMASI

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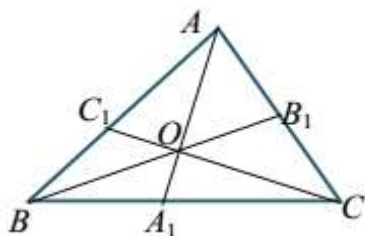
Annotatsiya: Ushbu ilmiy maqola akademik litseylar geometriya darslarida uchburchaklarni yechishda Cheva teoremasining qo'llanilishi haqidagi nazariy bilimni mustahkamlashda, matematika fanini bilish darajasini aniqlovchi Milly sertifikat olish uchun mustaqil tayyorlanuvchilarga yaqindan yordam beradi.

Kalit so'zlar: Uchburchak, nisbatlar, kesmalar yo'nalishi

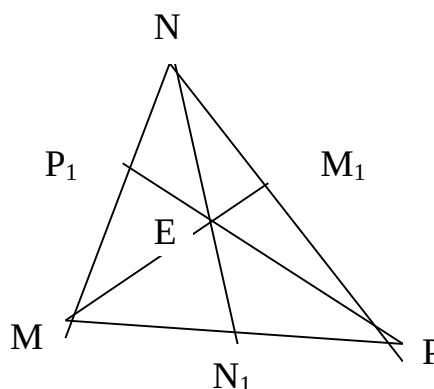
Ushbu maqola yoshlar uchun sifatli ta'limni ta'minlashda, iqtidorli yoshlar qobiliyatini rivojlantirishda, shuningdek, umumta'lim maktablari, akademik litseylarda faoliyat yuritayotgan aniq fanlar metod birlashmalarida o'rganib chiqilib, darslarda foydalanilsa, albatta, ta'lim sifati oshishida yordam beradi degan umiddamiz.

Cheva teoremasi (Isbotsiz). Agar O nuqta ABC uchburchakning ichida yotuvchi ixtiyoriy nuqta bo'lsa, AO , BO , CO to'g'ri chiziqlar BC , CA , AB tomonlarini mos ravishda A_1 , B_1 , C_1 nuqtalarda kesib o'tsa, ushbu munosabat o'rinlidir (1-rasm):

$$\frac{BA_1}{A_1C} \cdot \frac{CB_1}{B_1A} \cdot \frac{AC_1}{C_1B} = 1$$



1-rasm



2-rasm

Cheva teoremasiga oid masalalar

1-masala. MNP uchburchakning ichidagi ixtiyoriy E nuqta va uning uchlari orqali to'g'ri chiziqlar o'tkazilgan. Bu to'g'ri chiziqlar uchburchak tomonlarini mos ravishda M_1, N_1, P_1 nuqtalarda kesib o'tadi. Agar $MN_1:N_1P=2:3$, $PM_1:M_1N=3:4$ kabi bo'lsa, $NP_1:P_1M$ ni toping.

Yechilishi: Bu masala yechimini topishda Cheva teoremasi yordam beradi. Teoremani qo'llash uchun masala shartiga mos chizma chizamiz (2-rasm). Cheva teoremasiga ko'ra $\frac{MN_1}{N_1P} \cdot \frac{PM_1}{M_1N} \cdot \frac{NP_1}{P_1M} = 1$. Masala shartiga asosan $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{NP_1}{P_1M} = 1$. Bundan

$$\frac{NP_1}{P_1M} = 2. \text{ Javob: } 2$$

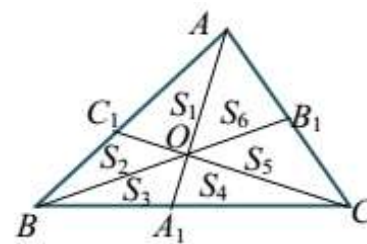
2-masala. ABC uchburchakning ichida yotuvchi ixtiyoriy O nuqta orqali o'tgan AO , BO , CO to'g'ri chiziqlar BC , CA , AB tomonlarini mos ravishda A_1 , B_1 , C_1



nuqtalarda kesib o'tadi, natijada uchburchak $S_1, S_2, S_3, S_4, S_5, S_6$ yuzli uchburchaklarga bo'linadi. Agar $BA_1:A_1C=2:3$ bo'lsa, $\frac{S_1+S_2}{S_5+S_6}$ ni toping.

Yechilishi: $\triangle AA_1B$ va $\triangle AA_1C$ uchburchaklar teng balandlikka ega bo'lgani uchun ularning yuzalar nisbati asoslar nisbatiga tengdir.

$$\frac{BA_1}{A_1C} = \frac{S_{\triangle AA_1B}}{S_{\triangle AA_1C}} = \frac{S_1+S_2+S_3}{S_4+S_5+S_6} \quad (1)$$



3-rasm.

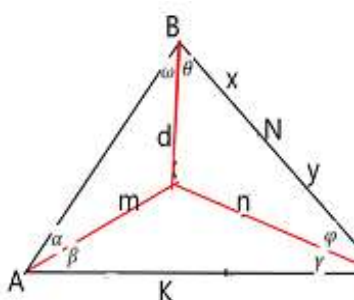
Shunga o'xshash $\triangle OA_1B$ va $\triangle OA_1C$ uchburchaklar yuzalar nisbati asoslar nisbatiga tengdir: $\frac{BA_1}{A_1C} = \frac{S_{\triangle OA_1B}}{S_{\triangle OA_1C}} = \frac{S_3}{S_4}$. (2)

Bu so'nggi ikkita proporsiyadan quyidagi hosilaviy proporsiya kelib chiqadi. (1) va (2) tengliklar teng nisbatlar bo'lgani uchun ularni tenglashtiramiz va hosil bo'lgan proporsiyani bajaramiz.

$$\frac{S_1+S_2+S_3}{S_4+S_5+S_6} = \frac{S_3}{S_4} \text{ bundan } (S_1+S_2) \cdot S_4 = S_4 \cdot S_3 + (S_4+S_5) \cdot S_3$$

$$\frac{BA_1}{A_1C} = \frac{S_{\triangle AA_1B}}{S_{\triangle AA_1C}} = \frac{S_1+S_2+S_3}{S_4+S_5+S_6} = \frac{S_1+S_2+S_3-S_3}{S_4+S_5+S_6-S_4} = \frac{S_1+S_2}{S_5+S_6} = \frac{2}{3}. \text{ Javob: } \frac{2}{3}$$

3-masala. ABC uchburchakda $\beta = 30^\circ, \gamma = 30^\circ, \varphi = 45^\circ, \omega = 30^\circ$ bo'lsa, $\sin \theta \cdot \sin \alpha$ ni toping.



Yechilishi: Uchburchaklar burchaklari uchun sinuslar teoremasini qo'llaymiz:

$$\begin{cases} \frac{n}{\sin \beta} = \frac{m}{\sin \gamma} \\ \frac{n}{\sin \theta} = \frac{d}{\sin \varphi} \\ \frac{d}{\sin \alpha} = \frac{m}{\sin \omega} \end{cases} \Rightarrow \begin{cases} \frac{m}{n} = \frac{\sin \gamma}{\sin \beta} \\ \frac{n}{d} = \frac{\sin \theta}{\sin \varphi} \\ \frac{d}{m} = \frac{\sin \alpha}{\sin \omega} \end{cases}$$

Bundan Cheva teoremasiga ko'ra $\frac{m}{n} \cdot \frac{n}{d} \cdot \frac{d}{m} = \frac{\sin 30^\circ}{\sin 30^\circ} \cdot \frac{\sin \theta}{\sin 45^\circ} \cdot \frac{\sin \alpha}{\sin 30^\circ} = 1$. Demak,

$$\sin \theta \cdot \sin \alpha = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}. \text{ Javob: } \frac{\sqrt{2}}{4}$$

Mustaqil yechish uchun masalalar

1. ABC uchburchakning ichidagi ixtiyoriy O nuqta va uning uchlari orqali to'g'ri chiziqlar o'tkazilgan. Bu to'g'ri chiziqlar uchburchak tomonlarini mos ravishda A_1, B_1, C_1 nuqtalarda kesib o'tadi. Agar $BA_1:A_1C=3:4, AC_1:C_1B=4:5$ kabi bo'lsa, $CB_1:B_1A$ ni toping. Javob: $\frac{5}{3}$

2. ABC uchburchakning ichidagi ixtiyoriy O nuqta va uning uchlari orqali to'g'ri chiziqlar o'tkazilgan. Bu to'g'ri chiziqlar uchburchak tomonlarini mos ravishda A_1, B_1, C_1 nuqtalarda kesib o'tadi. Agar $AC_1:C_1B=3:2, BA_1:A_1C=4:3$ kabi bo'lsa, $AB_1:B_1C$ ni toping. Javob: 2

3. ABC uchburchakning ichida yotuvchi ixtiyoriy O nuqta orqali o'tgan AO, BO, CO to'g'ri chiziqlar BC, CA, AB tomonlarini mos ravishda A_1, B_1, C_1



nuqtalarda kesib o'tadi. Agar $\frac{AC_1}{C_1B} = \frac{2}{5}$ va $S_{\Delta OA_1C} = 3$, $S_{\Delta OB_1C} = 7$, $S_{\Delta OB_1A} = 5$ berilgan bo'lsa, $S_{\Delta OA_1B}$ ni toping. Javob: 27

FOYDALANILGAN ADABIYOTLAR:

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2. Israilov I., Pashayev Z. Geometriya. Akademik litseylar uchun o'quv qo'llanma. I qism. – T.: O'qituvchi, 2004. Israilov I., Pashayev Z. Geometriyadan masalalar to'plami. Akademik litseylar uchun o'quv qo'llanma. – T.: O'qituvchi, 2001.