UCHBURCHAKLARNI YECHISHDA MENELAY TEOREMASI

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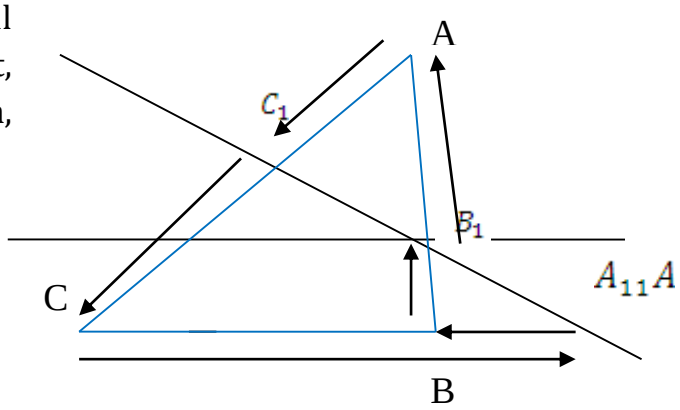
Annotatsiya: *Ushbu ilmiy maqola akademik litseylar geometriya darslarida uchburchaklarni yechishda Menelay teoremasining qo‘llanilishi haqidagi nazariy bilimni mustahkamlashda yaqindan yordam beradi.*

Kalit so‘zlar: *Nisbatlar, kesmalar yo‘nalishi*

Ushbu maqola yoshlar uchun sifatli ta’limni ta’minlashda, iqtidorli yoshlar qobiliyatini rivojlantirishda, shuningdek, umumta’lim maktablari , akademik litseylarda faoliyat yuritayotgan aniq fanlar metod birlashmalarida o‘rganib chiqilib, darslarda foydalanilsa, albatta, ta’lim sifati oshishida yordam beradi degan umiddamiz.

Menelay teoremasi(isbotsiz). Agar biror to‘g‘ri chiziq ABC uchburchakning AB, BC, CA tomonlarini yoki ularning davomlarini C_1,A_1, B_1 nuqtalarda kesib o‘tsa, u holda $\frac{CA_1}{A_1B} \cdot \frac{BB_1}{B_1A} \cdot \frac{AC_1}{C_1C} = -1$ tenglik o‘rinli bo‘ladi.

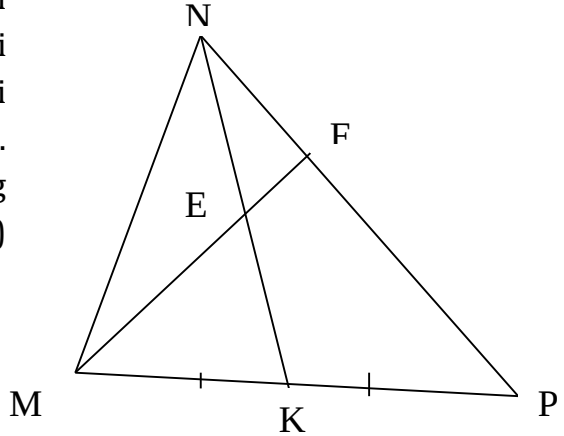
(Eslatma: Agar har bir nisbatni tashkil etuvchi kesmalar bir xil yo‘nalishga ega bo‘lsa, nisbat musbat, qarama- qarshi yo‘nalishga ega bo‘lsa, manfiy hisoblanadi.)



Menelay teoremasiga oid masalalar

1-masala. MNP uchburchakning NK medianasida E nuqta shunday olinganki, NE:EK=3:4 kabi bo‘lgan. ME to‘g‘ri chiziq NP tomonni F nuqtada kesadi. N uchidan boshlab hisoblaganda F nuqta NP tomonni qanday nisbatda bo‘ladi?

Yechilishi: Bu masala Menelay teoremasi yordamida osongina yechiladi. Bu teoremani qo‘llash uchun diqqatni NKP uchburchakni MF to‘g‘ri chiziq kesib o‘tgan deb olamiz. Shunda bu to‘g‘ri chiziq NKP uchburchakning KP tomonining davomida (M nuqtada) kesgan hisoblanadi.



Menelay teoremasiga ko‘ra $\frac{PM}{MK}\cdot\frac{KE}{EN}\cdot\frac{NF}{FP}=-1$. Natijada nisbat manfiy chiqmasligini ta’minlash uchun MK kesmaning yo‘nalishini o‘zgartirib yozamiz: $\frac{PM}{KM}\cdot\frac{KE}{EN}\cdot\frac{NF}{FP}=1$. Bundan NK mediana bo‘lganligi sababli PM=2·KM.

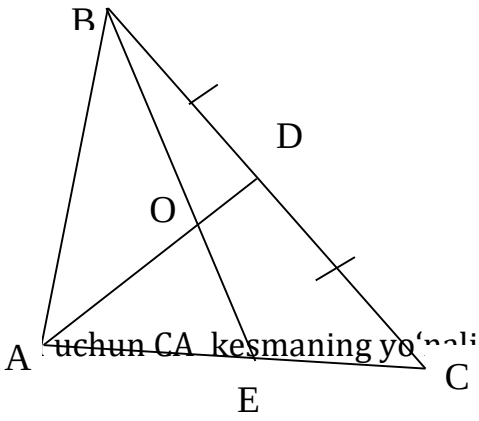
$\frac{2\cdot KM}{KM}\cdot\frac{4}{3}\cdot\frac{NF}{FP}=1$. Demak, $\frac{NF}{FP}=\frac{3}{8}$. Javob: $\frac{3}{8}$

2-masala. ABC uchburchakning AC tomonida E nuqta shunday olinganki, AE:EC=3:2 kabi bo‘lgan.

a) AD mediana BE kesmani qanday nisbatda bo‘ladi ?

b) BE kesma AD medianani A uchidan boshlab hisoblaganda qanday nisbatda bo‘ladi?

Yechilishi: a) Bunda diqqatni BEC uchburchakni AD to‘g‘ri chiziq kesib o‘tgan deb olamiz. Shunda bu to‘g‘ri chiziq BEC uchburchakning EC tomonini uning davomidagi A nuqtada kesgan hisoblanadi.



Menelay teoremasiga ko‘ra

$$\frac{CA}{AE}\cdot\frac{EO}{OB}\cdot\frac{BD}{DC}=-1$$

Natijada nisbat manfiy chiqmasligini ta’min uchun CA kesmaning yo‘nalishini o‘zgartirib yozamiz:

$\frac{AC}{AE}\cdot\frac{EO}{OB}\cdot\frac{BD}{DC}=1$. Bundan AD mediana bo‘lganligi sababli BD=DC hamda AC=AE+EC

$\frac{5x}{3x}\cdot\frac{EO}{OB}\cdot\frac{DC}{DC}=1$. Demak, $\frac{EO}{OB}=\frac{3}{5}$.

Javob: $\frac{3}{5}$

b) BE kesma AD medianani qanday nisbatda bo‘lishini topish uchun diqqatni ACD uchburchakni BE to‘g‘ri chiziq kesib o‘tgan deb qaraymiz. Shunda bu to‘g‘ri chiziq ACD uchburchakning CD tomonini uning davomidagi B nuqtada kesgan hisoblanadi. Menelay teoremasiga ko‘ra $\frac{CB}{BD}\cdot\frac{DO}{OA}\cdot\frac{AE}{EC}=-1$

Nisbat manfiy chiqmasligini ta’minlash uchun CB kesmaning yo‘nalishini o‘zgartirib yozamiz: $\frac{BC}{BD}\cdot\frac{DO}{OA}\cdot\frac{AE}{EC}=1$

Bundan AD mediana bo‘lganligi sababli BD=DC. Demak , BC=2·BD.

$$\frac{2\cdot BD}{BD}\cdot\frac{DO}{OA}\cdot\frac{3}{2}=1$$

$\frac{DO}{OA}=\frac{1}{3}$. BE kesma AD medianani A uchidan boshlab hisoblaganda esa $\frac{AO}{OD}=\frac{3}{1}$

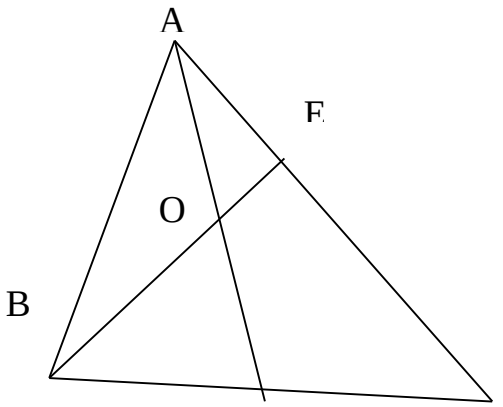
Javob: 3

3-masala. ABC uchburchakning AC va BC tomonlarida , mos ravishda , E va D nuqtalar shunday olinganki, AE:EC=2:3, BD:DC=3:1 kabi bo‘lgan.

a) BE kesma AD ni qanday nisbatda bo‘ladi ?

b) AD kesma BE kesmani B uchidan boshlab hisoblaganda qanday nisbatda bo‘ladi?

Yechilishi: a) Diqqatni ADC uchburchakni BE to‘g‘ri chiziq kesib o‘tgan deb tushunamiz. Bu to‘g‘ri chiziq ADC uchburchakning DC tomonini uning davomidagi B nuqtada kesgan hisoblanadi.



Menelay teoremasiga ko‘ra

$$\frac{CB}{BD}\cdot\frac{DO}{OA}\cdot\frac{AE}{EC}=-1$$

Nisbat manfiy bo‘lishini ta’minlash uchun CB kesmaning D nalishini o‘zga ib yozamiz: $\frac{BC}{BD}\cdot\frac{DO}{OA}\cdot\frac{AE}{EC}=1$. Masala shartiga ko‘ra

$\frac{4y}{3y}\cdot\frac{DO}{OA}\cdot\frac{2x}{3x}=1$. Demak, $\frac{DO}{OA}=\frac{9}{8}$. Bundan $\frac{AO}{OD}=\frac{8}{9}$.

Javob: $\frac{8}{9}$

b) AD kesma BE kesmani qanday nisbatda bo‘lishini topish uchun diqqatni BCE uchburchakni AD to‘g‘ri chiziq kesib o‘tgan deb qaraymiz. AD to‘g‘ri chiziq BCE uchburchakning CE tomonini uning davomidagi A nuqtada kesgan hisoblanadi. Menelay teoremasiga ko‘ra

$$\frac{CA}{AE}\cdot\frac{EO}{OB}\cdot\frac{BD}{DC}=-1$$

Nisbat manfiy chiqmasligini ta’minlash uchun CA kesmaning yo‘nalishini o‘zgartirib yozamiz: $\frac{AC}{AE}\cdot\frac{EO}{OB}\cdot\frac{BD}{DC}=1$ Bundan AC=5x ekanligidan $\frac{5x}{2x}\cdot\frac{EO}{OB}\cdot\frac{3y}{y}=1$

$\frac{EO}{OB}=\frac{2}{15}$. AD kesma BE kesmani B uchidan boshlab hisoblaganda esa $\frac{EO}{OB}=\frac{15}{2}$

Javob: 7,5

FOYDALANILGAN ADABIYOTLAR:

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