ZINAPOYASIMON GRAFLARDA TO’LQIN TARQALISH TENGLAMASI UCHUN BOSHLANG’ICH – CHEGARAVIY MASALA

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Annotatsiya: *Ushbu maqolada zinapoyasimon graflarda to’lqin tarqalish tenglamasi uchun boshlang’ich-chegaraviy masala o’rganildi. Qo’yilgan masala o’zgaruvchilarni ajratish usuli yordamida yechildi va yechimning mavjudligi va yagonaligi aniqlandi.*

Kalit so’zlar: *metrik graf, zinapoyasimon graf, o’zgaruvchilarni ajratish usuli, aprior baholash*

Аннотация: *В данной работе исследуется начально-краевая задача для уравнения распространения волна лестничных графах. Данная задача решена методом Фурье и установлены существование и единственность решения.*

Ключевые слова: *метрический график, лестничная диаграмма, метод Фурье, априорная оценка.*

Annotation: *In this paper, we investigate local boundary value problems for the wave equation on ladder type metric graph. The main goal of this work is to study the uniqueness and the existence of solution of the formulated problem. Using by the method of separation of variables we find a solution of the investigated problem in the form of the Fourier series. Existence and uniqueness of solution of the considered problems are defined.*

Keywords: *Metric graph, ladder type metric graph, method of separation of variables, apriori estimates.*

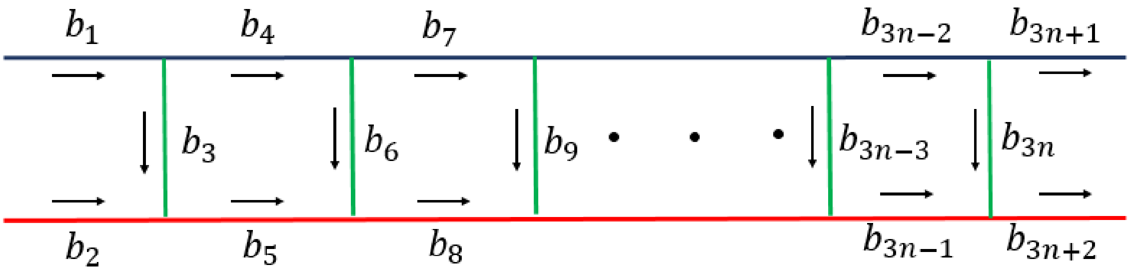
Kirish

Metrik graflarda differensial tenglamalar uchun boshlang’ich - chegaraviy masalalarni o‘rganish nisbatan yangi soha hisoblanib, asosan 1990-yillarga kelib grafda chiziqli Shroedinger tenglamasi (Kvant grafi) o‘rganila boshlangan[1]. Graflarda boshqa turdagi tenglamalar asosan 2010-yillarga kelib o‘rganila boshlangan. Bulardan chiziqsiz Shroedinger tenglamasi[2], Sinus – Gordon tenglamasi, Korteveg-De Friz tenglamalari bo‘yicha olingan natijalarni keltirishimiz mumkin. G. Berkolaiko o‘z ishlarida uch qirrali yulduzsimon graflarda turli fizik jarayonlarni tadqiq qilishda foydalangan[3]. Z.A. Sobirov va M.R. Eshimbetov esa zinapoyasimon graflarda issiqlik tarqalish jarayonlarini[4] o‘rgangan. Zinapoyasimon graflarda to‘lqin tarqalish tenglamasi uchun boshlang’ich-chegaraviy masala ushbu ishda birinchi marotaba qaralmoqda.

Zinapoyasimon graflarda to‘lqin jarayonlarini o‘rganish amaliy jihatdan muhim masalalardan hisoblanadi. Bunday graflar DNK, RNK molekulalarida kechadigan impuls tarqalishini tadqiq qilishda model vazifasini bajaradi.

1. Masalaning qo'yilishi

Bizga qirralari $3n+2$, $(n \in N)$ ta teng uzunliklarga ega bo'lgan kesmalarni birlashtirishdan hosil bo'lgan Γ zinapoyasimon graf berilgan bo'lsin (1-chizma). Grafning bog'lamlarini (qirralarini) b_k, $k=\overline{1,3n+2}$ ko'rinishda belgilaymiz. Har bir b_k bondda aniqlanadigan x_k koordinatalarni biz x deb qaraymiz hamda barcha b_k qirralarga $(0,L)$ kesmani mos qo'yamiz, ya'ni $b_k \sim (0,L)$, $k=\overline{1,3n+2}$.



(1-chizma)

Grafning har bir qirrasida quyidagi to'lqin tarqalish tenglamasini qaraymiz

$$u_{k\,tt}(x,t)=a^2u_{k\,xx}(x,t)+f_k(x,t),\ (x,t)\in\big((0,L)\times(0,T)\big),\quad k=\overline{1,3\text{n}+2}. \tag{1}$$

Bu yerda $u_k=(u_1,u_2,u_3,\dots,u_{3n+2})^T$, $f_k=(f_1,f_2,f_3,\dots,f_{3n+2})^T$. $f_k(x,t)$ berilgan ma'lum funksiyalar $k=\overline{1,3n+2}$.

Γ grafda (1) tenglama uchun quyidagi masalani qarab chiqamiz.

Masala 1. (1) ko'rinishida berilgan to'lqin tarqalish tenglamasining $b_k\times(0,T)$ sohada aniqlangan va quyidagi:

boshlang'ich

$$u_k(x,t)\big|_{t=0}=\varphi_k(x),\quad u_{k\,t}(x,t)\big|_{t=0}=\psi_k(x),\ 0\le x\le L,\ k=\overline{1,3\text{n}+2}, \tag{2}$$

bir jinsli chegaraviy

$$u_k(x,t)\big|_{x=0}=0,\ k\in\{1,2\},\quad u_k(x,t)\big|_{x=L}=0,\ k\in\{3n+1,3n+2\},\ t\ge 0, \tag{3}$$

hamda quyidagi ulanish shartlarini (Kirxgoff shartlari)

$$\begin{aligned} &u_{3k-2}(x,t)\big|_{x=L}=u_{3k+1}(x,t)\big|_{x=0}=u_{3k}(x,t)\big|_{x=0},\\ &u_{3k-2,x}(x,t)\big|_{x=L}=u_{3k+1,x}(x,t)\big|_{x=0}+u_{3k,x}(x,t)\big|_{x=0}\ , \end{aligned} \tag{4}$$

$$\begin{aligned} &u_{3k}(x,t)\big|_{x=L}=u_{3k-1}(x,t)\big|_{x=L}=u_{3k+2}(x,t)\big|_{x=0},\\ &u_{3k,x}(x,t)\big|_{x=L}+u_{3k-1,x}(x,t)\big|_{x=L}=u_{3k+2,x}(x,t)\big|_{x=0}\ ,\ k=\overline{1,n},\ t\ge 0 \end{aligned} \tag{5}$$

qanoatlantiruvchi $u_k=(u_1,u_2,u_3,\dots,u_{3n+2})^T$ yechim topilsin.

Bu yerda $\varphi_k(x)$ va $\psi_k(x)$, $k=\overline{1,3n+2}$ funksiyalar berilgan, yetarlicha silliq funksiyalar. Bundan tashqari, (3)-(5) shartlarga mos quyidagi xossalarga ega bo'lsin deb qaraymiz:

$$\varphi_k(0)=0,\ \psi_k(0)=0,\quad k\in\{1,2\},$$
$$\varphi_k(L)=0,\ \psi_k(L)=0,\quad k\in\{3n+1,3n+2\}, \tag{6}$$

$$\begin{aligned}&\varphi_{3k-2}(L)=\varphi_{3k+1}(0)=\varphi_{3k}(0),\\ &\varphi'_{3k-2}(L)=\varphi'_{3k+1}(0)+\varphi'_{3k}(0),\\ &\varphi_{3k}(L)=\varphi_{3k-1}(L)=\varphi_{3k+2}(0),\\ &\varphi'_{3k}(L)+\varphi'_{3k-1}(L)=\varphi'_{3k+2}(0),\quad k=\overline{1,n},\end{aligned} \tag{7}$$

$$\begin{aligned}&\psi_{3k-2}(L)=\psi_{3k+1}(0)=\psi_{3k}(0),\\ &\psi'_{3k-2}(L)=\psi'_{3k+1}(0)+\psi'_{3k}(0),\\ &\psi_{3k}(L)=\psi_{3k-1}(L)=\psi_{3k+2}(0),\\ &\psi'_{3k}(L)+\psi'_{3k-1}(L)=\psi'_{3k+2}(0),\quad k=\overline{1,n}.\end{aligned} \tag{8}$$

2. Metrik graflarda spektral masala

Biz (1) tenglamalarning bir jinsli qismini o'zgaruvchilarni ajratish usuli (Furye usuli)dan foydalanib yechamiz

$$X''_k(x)+\lambda^2 X_k(x)=0,\quad k=\overline{1,3n+2}, \tag{9}$$

(3)-(5) shartlarni qanoatlantirsak,

$$X_j(0)=0,\ \ j\in\{1,2\},\quad X_l(L)=0,\ \ l\in\{3n+1,3n+2\} \tag{10}$$

$$X_{3k-2}(L)=X_{3k+1}(0)=X_{3k}(0), \tag{11}$$

$$X'_{3k-2}(L)-X'_{3k+1}(0)-X'_{3k}(0)=0, \tag{12}$$

$$X_{3k}(L)=X_{3k-1}(L)=X_{3k+2}(0), \tag{13}$$

$$X'_{3k}(L)+X'_{3k-1}(L)-X'_{3k+2}(0)=0,\ k=\overline{1,n} \tag{14}$$

munosabatlarga ega bo'lamiz.

Odatda (9)-(14) ko'rinishdagi masalalarga **Shturm-Liuvill spektral masalasi** deyiladi. Metrik graflarda spektral masalalarni [5]-[6] adabiyotlarda uchratish mumkin.

Differensial tenglamalar kursidan ma'lumki, (9) tenglamaning umumiy yechimi quyidagicha

$$X_k(x)=c_k\cos\lambda x+d_k\sin\lambda x \tag{15}$$

ko'rinishga ega. (15) tenglamaning har bir qirradagi c_k va d_k noma'lum koeffitsiyentlarga nisbatan (10)-(14) shartlardan foydalanib,

$$M(\lambda)Q=0 \tag{16}$$

tenglamalar sistemasiga ega bo‘lamiz. Bu yerda Q noma’lum koeffitsiyentlardan tuzilgan vektor bo‘lib quyidagi ko‘rinishda yoziladi:

$$Q=(c_1,d_1,c_2,d_2,...,c_{3n+2},d_{3n+2})^T .$$

(10)-(14) shartlarni e’tiborga olsak, bizda $M(\lambda)$ matritsa quyidagicha ko‘rinishga ega bo‘ladi.

$$M_{6n+2\;6n+2}=\begin{pmatrix} A_{6\;0} & B_{6\;4} & C_{6\;4} & A_{6\;6(n-1)} \\ A_{6\;4} & D_{6\;6} & C_{6\;4} & A_{6\;6(n-2)} \\ A_{6\;10} & D_{6\;6} & C_{6\;4} & A_{6\;6(n-3)} \\ \dots & \dots & \dots & \dots \\ A_{6\;4+(n-2)6} & D_{6\;6} & C_{6\;4} & A_{6\;0} \\ 0 & 0 & 0 & E_{2\;4} \end{pmatrix}$$

Aytishimiz mumkinki, $M_{6n+2\;6n+2}(\lambda)$ matritsa $B_{6\;4}$, $D_{6\;6}$ hamda $E_{2\;4}$ matritsalarga nisbatan diagonal matritsadir. Bu yerda, A_{ij} matritsalar nol matritsalardir. Agar qulaylik uchun $\cos\lambda L$ va $\sin\lambda L$ qiymatlar uchun $a=\cos\lambda L$, $b=\sin\lambda L$ belgilashlarni kiritsak, $B_{6\;4}$, $C_{6\;4}$, $D_{6\;6}$ hamda $E_{2\;4}$ matritsalar quyidagi ko‘rinishlarda ifodalanadi:

$$B_{6\;4}=\begin{pmatrix} b & 0 & -1 & 0 \\ b & 0 & 0 & 0 \\ -a & 0 & 0 & 1 \\ 0 & -b & a & b \\ 0 & 0 & a & b \\ 0 & -a & b & -a \end{pmatrix}, \qquad C_{6\;4}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$D_{6\;6}=\begin{pmatrix} a & b & 0 & 0 & -1 & 0 \\ a & b & 0 & 0 & 0 & 0 \\ b & -a & 0 & 0 & 0 & 1 \\ 0 & 0 & a & b & -a & -b \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & b & -a & b & -a \end{pmatrix}, \qquad E_{2\;4}=\begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \end{pmatrix}.$$

Biz quyida $n=1$ bo‘lgan hol uchun bitta misol keltiramiz. Ya’ni qirralari soni 5 ta bo‘lgan graf uchun qaraymiz. (15) tenglik va (3) - (5) shartlarga ko‘ra

$$\sin^3\lambda L\left(8\cos^2\lambda L-\sin^2\lambda L\right)=0,\; \lambda_{1,m}=\lambda_{2,m}=\lambda_{3,m}=\frac{\pi m}{L},\; m\in N$$

$$\lambda_{4,m}=\lambda_{5,m}=\pm\frac{1}{2L}\arccos\left(-\frac{7}{9}\right)+\frac{\pi m}{L},\; m\in N$$

ko‘rinishdagi $\lambda_{k,m}$ $(k=\overline{1,5})$ xos sonlarga ega bo‘lamiz. (15) tenglikni inobatga olib bu xos sonlarga mos bo‘lgan $X_{km}(x)$ $(k=\overline{1,5})$ xos funksiyalarni yozamiz

$$X_{1m}(x)=c_1\cos\frac{\pi m}{L}x+d_1\sin\frac{\pi m}{L}x,\ X_{2m}(x)=c_2\cos\frac{\pi m}{L}x+d_2\sin\frac{\pi m}{L}x,$$

$$X_{3m}(x)=c_3\cos\frac{\pi m}{L}x+d_3\sin\frac{\pi m}{L}x,\ \ X_{4m}(x)=c_4\cos\lambda_{4,m}x+d_4\sin\lambda_{4,m}x,$$

$$X_{5m}(x)=c_5\cos\lambda_{5,m}x+d_5\sin\lambda_{5,m}x.$$

3. Masala yechimini tadqiq qilish: yechimning mavjudligi va yagonaligi

Teorema 1. Agar $\varphi_k(x)$, $\psi_k(x)\in C^1[0,L]$, $\frac{\partial}{\partial x}f_k(x,t)\in C\big([0,L]\times(0,T)\big)$ $k=\overline{1,3n+2}$ sinflarda aniqlangan funksiyalar bo‘lib, $[0,L]$ intervallarda $\varphi_k''(x)$, $\psi_k''(x)$ va $\big((0,L)\times(0,T)\big)$ sohada $\frac{\partial^2}{\partial x^2}f_k(x,t)$ funksiyalar absolyut integrallanuvchi bo‘lsa, bundan tashqari $\varphi_k(x)$ funksiyalar uchun

$$\varphi_k''(0)=0,\qquad k\in\{1,2\},\qquad \varphi_k''(L)=0,\qquad k\in\{3n+1,3n+2\},$$

$$\varphi_{3k}''(L,t)=\varphi_{3k-2}''(0,t)=\varphi_{3k+1}''(0,t),\ k=\overline{1,n},$$

$$\varphi_{3k}''(L,t)=\varphi_{3k-1}''(L,t)=\varphi_{3k+2}''(0,t),\ k=\overline{1,n}$$

shartlar o‘rinli hamda (3)-(5) va (6)-(8) shartlar mos ravishda $\psi_k(x)$, $f_k(x,t)$, $k=\overline{1,3n+2}$ funksiyalar uchun ham o‘rinli bo‘lsa, (1)–(5) masalaning yechimi mavjud va yagonadir.

ISBOT. Berilgan $f(x,t)$ funksiyalarni xos funksiyalar yordamida Furye qatoriga yoyamiz.

$$f(x,t)=\sum_{m=0}^{\infty}f_m(t)X_m(x)\qquad (17)$$

Bu yerda $f_m(t)$ (17) Furye qatorining koeffitsiyentlari. (1) tenglamaning yechimini quyidagi ko‘rinishda yozsak,

$$u(x,t)=\sum_{m=0}^{\infty}X_m(x)W_m(t),\ k=\overline{1,3n+2}\qquad (18)$$

Ushbu yechimdan hamda (1) tenglamadan foydalanib

$$\sum_{m=0}^{\infty}(W_m''(t)+a^2\lambda_m{}^2W_m(t)-f_m(t))X_m(x)=0\qquad (19)$$

tenglikni hosil qilamiz. $X_m(x)$ funksiya qaralayotgan masalaning xos funksiyasi ekanligidan, ushbu

$$W_m''(t)+a^2\lambda_m^2W_m(t)=f_m(t)\qquad (20)$$

bir jinsli bo‘lmagan differensial tenglamaga ega bo‘lamiz. Ma’lumki, (20) tenglamaning umumiy yechimi

$$W_m(t)=\frac{1}{a\lambda_m}\int_0^t \sin a\lambda_m(t-z) f_m(z)dz + c_{1m}\cos a\lambda_m t + c_{2m}\sin a\lambda_m t \quad (21)$$

ko‘rinishida ifodalanadi[9]. Demak, (18) ga asosan (1) tenglamaning umumiy yechimini quyidagicha yozishimiz mumkin:

$$u_k(x,t)=\sum_{m=0}^{\infty}\left[\frac{1}{a\lambda_m}\int_0^t \sin a\lambda_m(t-z) f_m(z)dz + c_{1m}\cos a\lambda_m t + c_{2m}\sin a\lambda_m t\right]X_{km}(x)$$

$k=\overline{1,3n+2}$. (22)

Ma’lumki, berilgan $\varphi_k(x)$, $\psi_k(x)$ funksiyalarni quyidagi

$$\varphi_k(x)=\sum_{m=0}^{\infty}\varphi_m X_{km}(x)\ ,\quad \psi_k(x)=\sum_{m=0}^{\infty}\psi_m X_{km}(x) \quad (23)$$

ko‘rinishida Furye qatorlariga yoyishimiz mumkin. Bu yerda φ_m va ψ_m (23) Furye qatorining koeffitsiyentlaridir. Hosil qilingan (22) umumiy yechimiga (2) boshlang’ich shartni qanoatlantirsak,

$$c_{1m}=\varphi_m\ ,\quad c_{2m}=\frac{\psi_m}{a\lambda_m} \quad (24)$$

tengliklarga ega bo‘lamiz.

Demak, (22) va (24) tengliklarga asosan (1) tenglamaning (2) shartlarini qanoatlantiruvchi yechimi quyidagicha topilar ekan:

$$u_k(x,t)=$$

$$=\sum_{m=0}^{\infty}\left[\frac{1}{a\lambda_m}\int_0^t \sin a\lambda_m(t-z) f_m(z)dz + \varphi_m\cos a\lambda_m t + \frac{\psi_m}{a\lambda_m}\sin a\lambda_m t\right]X_{km}(x).$$

(25)

(23) tengliklarni berilgan $\varphi_k(x)$, $\psi_k(x)$ funksiyalar $\varphi_k(x)$, $\psi_k(x)\in L_2[0,L]$ ekanidan

$$\varphi_m=\sum_k\int_0^L \varphi_k(x)X_{km}(x)dx,\quad \psi_m=\sum_k\int_0^L \psi_k(x)X_{km}(x)dx \quad (26)$$

ko‘rinishda yozishimiz mumkin. (26) tengliklarning o‘ng tarafini mos ravishda ikki va uch marta bo‘laklab integrallab, (6)-(8) shartlarni hamda teorema shartlarini e’tiborga olsak,

$$\varphi_m=\frac{1}{\lambda_m^3}\sum_{k=1}^{3n+2}\int_0^L \varphi_k'''(x)X_{km}^*(x)dx,\quad \psi_m=-\frac{1}{\lambda_m^2}\sum_{k=1}^{3n+2}\int_0^L \psi_k''(x)X_{km}(x)dx. \quad (27)$$

tengliklarga ega bo‘lamiz, bu yerda $X_{km}^*(x)=c_k\sin\lambda_m x-d_k\cos\lambda_m x$.

Xuddi, $\varphi_k(x)$, $\psi_k(x)$ funksiyalarga o'xshash, (17) tenglikda berilgan $f_k(x,t)$ funksiyani ham $L_2[0,L]$ sinfga tegishliligidan, teoreme shartlaridan

$$f_m(t)=\sum_k\int_0^L f_k(x,t)X_{km}(x)dx=-\frac{1}{\lambda_m^2}\sum_{k=1}^{3n+2}\int_0^L\frac{d^2}{dx^2}f_k(x,t)X_{km}(x)dx \qquad (28)$$

tenglikka ega bo'lamiz.

Ma'lumki, (25) ko'rinishda topilgan funksiya qo'yilgan masala yechimi bo'lishi uchun, bu funksiya (1) tenglamani ham qanoatlantirishi kerak. Demak qaralayotgan sohada $u_k(x,t)$, $u_{k\,xx}(x,t)$, $u_{k\,tt}(x,t)$ funksiyalarning aniqlanganligini isbotlash talab etiladi.

Lemma 1. $X_{km}(x)=c_k\sin\lambda_m x+d_k\cos\lambda_m x$ xos funksiya uchun quyidagi tengsizlik o'rinli.

$$\left|X_{km}(x)\right|=\left|c_{km}\cos\lambda_m x+d_{km}\sin\lambda_m x\right|\le\sqrt{\frac{2}{L}}\ . \qquad (29)$$

Demak, (25) yechimni quyidagi ko'rinishda yozishimiz mumkin.

$$u_k(x,t)=\sum_{m=0}^{\infty}\left[\frac{M_1}{a\lambda_m^3}\int_0^t\sin a\lambda_m(t-z)dz+\frac{M_2}{\lambda_m^3}\cos a\lambda_m t+\frac{M_3}{a\lambda_m^3}\sin a\lambda_m t\right]\times$$
$$\times\sqrt{c_{km}^2+d_{km}^2}\sin\left(\lambda_m x+A_{km}\right)$$

Bu yerda M_1, M_2, M_3-o'zgarmas sonlar va $\cos A_{km}=\dfrac{d_{km}}{\sqrt{c_{km}^2+d_{km}^2}}$, $\sin A_{km}=\dfrac{c_{km}}{\sqrt{c_{km}^2+d_{km}^2}}$.

Lemma 2. Agar quyidagi

$$\max\left|\frac{M_1}{a}\sqrt{c_{km}^2+d_{km}^2}\cdot\int_0^t\sin a\lambda_m(t-z)dz\right|=M_{12}<+\infty,$$

$$\max\left|M_2\sqrt{c_{km}^2+d_{km}^2}\cdot\cos a\lambda_m t\right|=M_{22}<+\infty,$$

$$\max\left|\frac{M_3}{a}\sqrt{c_{km}^2+d_{km}^2}\cdot\sin a\lambda_m t\right|=M_{32}<+\infty,$$

shartlar o'rinli bo'lsa, (25) qator hamda uning x va t o'zgaruvchilar bo'yicha ikki marta hosila olish orqali hosil qilingan $u_{k\,xx}(x,t)$, $u_{k\,tt}(x,t)$ qatorlar ham qaralayotgan sohada tekis yaqinlashuvchi bo'ladi. Bu yerda M_{12}, M_{22}, M_{32} - o'zgarmas sonlar.

Lemma 2 ning isbotini bilamiz [8]. Bundan yechimga mos keladigan barcha cheksiz qatorlar va ular hosilalarining yaqinlashuvchiligini isbotladik. Bu esa qo'yilgan masala yechimi mavjudligini bildiradi.

Lemma 3. Qaralayotgan masalaning $u_k(x,t)$ yechimi uchun quyidagi tengsizlik o'rinli.

$$\int_0^t \|u_\tau(x,\tau)\|_\Gamma^2 d\tau \le h_1 \int_0^t (t-\tau)\| f(x,\tau)\|_\Gamma^2 d\tau + h_2\|\psi(x)\|_\Gamma^2 + h_3\|\varphi_x(x)\|_\Gamma^2$$

Bu yerda $h_i, i=1,2,3$ lar o'zgarmas sonlar.

ISBOT. (1) tenglamani $u_{kt}(x,t)$ ga skalyar ko'paytiramiz, Γ grafda skalyar ko'paytma ta'rifiga ko'ra

$$\left(u_{kt}(x,t),u_{tt}(x,t)\right)_\Gamma - \left(u_{kt}(x,t),a^2u_{xx}(x,t)\right)_\Gamma = \left(u_{kt}(x,t),f(x,t)\right)_\Gamma,$$

$$\sum_k \int_0^L u_{kt}(x,t)u_{ktt}(x,t)dx - a^2\sum_k\int_0^L u_{kt}(x,t)u_{kxx}(x,t)dx =$$

$$=\sum_k\int_0^L u_{kt}(x,t)f_k(x,t)dx. \qquad (30)$$

(30) tenglik masala shartlariga ko'ra quyidagi ko'rinishga ega bo'ladi:

$$\frac{1}{2}\frac{d}{dt}\|u_t(x,t)\|_\Gamma^2 + \frac{a^2}{2}\frac{d}{dt}\|u_x(x,t)\|_\Gamma^2 \le \frac{1}{2}\|u_t(x,t)\|_\Gamma^2 + \frac{1}{2}\|f(x,t)\|_\Gamma^2. \qquad (31)$$

Endi olingan natijani 0 dan t oraliqda τ bo'yicha integrallasak,

$$\|u_t(x,t)\|_\Gamma^2 + a^2\|u_x(x,t)\|_\Gamma^2 \le \int_0^t \left(\|u_\tau(x,\tau)\|_\Gamma^2 + \|f(x,\tau)\|_\Gamma^2\right)d\tau +$$

$$+\|u_t(x,0)\|_\Gamma^2 + a^2\|u_x(x,0)\|_\Gamma^2.$$

ko'rinishdagi tengsizlikka ega bo'lamiz. (2) shartni e'tiborga olsak,

$$\|u_t(x,t)\|_\Gamma^2 + a^2\|u_x(x,t)\|_\Gamma^2 \le$$

$$\le \int_0^t \left(\|u_\tau(x,\tau)\|_\Gamma^2 + \|f(x,\tau)\|_\Gamma^2\right)d\tau + \|\psi(x)\|_\Gamma^2 + a^2\|\varphi_x(x)\|_\Gamma^2. \qquad (32)$$

Bizga ma'lumki, (32) tengsizlik quyidagi tengsizliklarga teng kuchli:

$$a^2\|u_x(x,t)\|_\Gamma^2 \le \int_0^t \|u_\tau(x,\tau)\|_\Gamma^2 d\tau + \int_0^t \|f(x,\tau)\|_\Gamma^2 d\tau + c_1\|\psi(x)\|_\Gamma^2 + c_2\|\varphi_x(x)\|_\Gamma^2,$$

$$\|u_t(x,t)\|_\Gamma^2 \le \int_0^t \|u_\tau(x,\tau)\|_\Gamma^2 d\tau + \int_0^t \|f(x,\tau)\|_\Gamma^2 d\tau + c_1\|\psi(x)\|_\Gamma^2 + c_2\|\varphi_x(x)\|_\Gamma^2. \qquad (33)$$

(33) tengsizlikda $y(t)=\int_0^t \|u_\tau(x,\tau)\|_\Gamma^2 d\tau$ deb, Gronwall-Bellman tengsizligidan foydalanib[7], $y(0)=0$ ekanidan

$$\int_0^t \|u_\tau(x,\tau)\|_\Gamma^2 d\tau \le e^t\left[\int_0^t ds\int_0^s \|f(x,\tau)\|_\Gamma^2 d\tau + \int_0^t \|\psi(x)\|_\Gamma^2 d\tau + \int_0^t \|\varphi_x(x)\|_\Gamma^2 d\tau\right] =$$

$$=h_1\int_0^t (t-\tau)\left\| f(x,\tau)\right\|_\Gamma^2 d\tau + h_2\left\|\psi(x)\right\|_\Gamma^2 + h_3\left\|\varphi_x(x)\right\|_\Gamma^2 \qquad (34)$$

tengsizlikni hosil qilamiz. h_1, h_2 va h_3 musbat o‘zgarmaslardir. Lemma 3 isbot bo‘ldi.

Standart usullardan foydalanib (1)-(5) masalaning yechimini yagona ekanligini isbotlash mumkin. Agar, (34) tengsizlikda $f_k(x,\tau)\equiv 0$, $\psi_k(x)\equiv 0$ va $\varphi_k(x)\equiv 0$ deb olsak, u holda biz ega bo‘lamizki, $u_k(x,t)\equiv 0$.

Demak, (1)-(5) masalaning yechimi mavjud va yagonaligi kelib chiqdi. Teorema 1 to‘liq isbotlandi.

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