TO'RTINCHI TARTIBLI MURAKKAB TURDAGI LAPLAS OPERATORI QATNASHGAN TENGLAMA UCHUN CHEGARAVIY MASALA

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Annotatsiya: *Maqolada to'rtinchi tartibli murakkab turdagi tenglamalar uchun chegaraviy masala qaralgan bo'lib, masalaning yechimining yagonaligi integral energiya usulida, yechimning mavjudligini isbotlashda integral tenglamalari nazariyasidan foydalanilgan.*

Kalit so'zlar: *Grin formulasi, yuqori tartibli tenglama, Laplas operatori, yechimning yagonaligi, yechimning mavjudligi, energiya integrali.*

1. KIRISH

Hozirgi kunda yuqori tartibli differensial tenglamalar turli fizik va texnik jarayonlarni modellashtirishda keng qo'llanilmoqda. Xususan, elastiklik nazariyasi, plastinkalar va qobiqlar nazariyasi, gidrodinamika hamda boshqa ko'plab amaliy masalalarda to'rtinchi tartibli differensial operatorlar muhim rol o'ynaydi. Shu sababli bunday tenglamalar uchun chegaraviy masalalarni o'rganish nazariy va amaliy jihatdan dolzarb ilmiy masalalardan biri hisoblanadi.

2. ADABIYOTLAR TAHLILI

Murakkab turdagi tenglamalarni o'rganish va ularni tahlil qilish usullarini yaratishda juda ko'p olimlar shu jumaladan M.S.Salohiddinov[1,2], T.D.Jo'rayev[3] va ularning shogirdlari katta hissa qo'shishgan.

Mazkur magistrlik dissertatsiya ishida ham to'rtinchi tartibli xususiy hosilali differensial tenglama uchun to'g'ri to'rtburchak sohada chegaraviy masala o'rganilgan. Tahlil qilingan tenglamalar ham haqiqiy hamda kompleks harakteristikalarga ega bo'lib, biror-bir klassik tipga tegishli emas.

3. Masalaning qo'yilishi. Ushbu

$$\frac{\partial^2}{\partial x^2}\Delta u + cu = f(x,y) \qquad (1)$$

tenglamani $D=\{(x,y):0<x<1,0<y<1\}$ sohada qaraylik. Bunda $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ - Laplas operatori, $c(x,y)$ va $f(x,y)$ lar berilgan funksiyalar. Bu sohaning uchlarini $A(0;0)$, $B(1;0)$, $C(1;1)$, $D(0;1)$ deb belgilaymiz.

Shunday $u(x,y)\in C^2(\overline{D})\cap C^4(D)$ funksiyani topingki, u (1) tenglamani va quyidagi (2)-(4) chegaraviy shartlarni qanoatlantirsin:

$$u(0,y)=\varphi_0(y) \qquad u(x,0)=\psi_0(x)$$
$$u(1,y)=\varphi_1(y) \qquad u(x,1)=\psi_1(x) \qquad (2)$$

$$u(x,-x+1)=\psi_2(x) \qquad (3)$$

$$u_x(0,y)=\varphi_3(y) \qquad (4)$$

Bu yerda $c(x,y)$ va φ_i, ψ_i $(i=0,3)$ lar berilgan funksiyalar va ularni berilgan sinfdan bo'lishini ta'minlovchi kelishuv shartlari o'rinli.

4. Masala yechimining yagonaligi.

Teorema. Agar $c(x,y)\geq 0 \quad (x,y)\in D, \quad \psi_1'(1)=\psi_0'(1)=0$ shartlar bajarilsa, u holda (1)-(4) masalaning bittadan ortiq yechimi mavjud emas.

Isbot. Aytaylik, (1) - (4) masalani $u_1(x,y)$ va $u_2(x,y)$lar qanoatlantirsin. U holda $u=u_1-u_2$ ham (1) tenglamani va

$$\varphi_i\equiv\psi_i\equiv f(x,y)\equiv 0, \quad i=0,1,2,3 \qquad (5)$$

bir jinsli chegaraviy shartlarni qanoatlantiradi.

D_ε bilan $D_\varepsilon=\{(x,y):\varepsilon<x<1-\varepsilon,\varepsilon<y<1-\varepsilon\}$ sohani belgilaymiz (Γ_ε - D_ε sohaning chegarasi). Berilgan (1) tenglamani $u(x,y)$ ga ko'paytirib, D_ε soha bo'yicha integrallaymiz:

$$\iint_{D_\varepsilon} u\left(\frac{\partial^2}{\partial x^2}\Delta u+cu\right)dxdy=0 \qquad (6)$$

Bunda $u\dfrac{\partial^2}{\partial x^2}\Delta u$ ifodani divergen holatga o'tkazaylik:

$$u\frac{\partial^2}{\partial x^2}\Delta u=u\frac{\partial}{\partial x}\Delta u_x=\frac{\partial}{\partial x}(u\Delta u_x)-u_xu_{xx}=\frac{\partial}{\partial x}(u\Delta u_x)-u_x(u_{xxx}+u_{yyy})=$$
$$=\frac{\partial}{\partial x}(u\Delta u_x)-\frac{\partial}{\partial x}(u_xu_{xx})+u_{xx}^2-\frac{\partial}{\partial y}(u_xu_{xy})+u_{xy}^2 \qquad (7)$$

(6) tenglikka (7) dan foydalanib, Grin formulalarini qo'llaymiz:

$$\int_{\Gamma_\varepsilon} u\Delta u_x dy-\int_{\Gamma_\varepsilon} u_xu_{xx}dy+\int_{\Gamma_\varepsilon} u_xu_{xy}dy+\iint_{D_\varepsilon}(u_{xx}^2+u_{xy}^2+c(x,y)u^2)dxdy=0 \qquad (8)$$

Bu yerda har bir integralni alohida hisoblaymiz:

$$J_1=\int_{\Gamma_\varepsilon} u\Delta u_x dy$$

$\varepsilon\to 0$ da (2) va (5) bir jinsli shartlarga asosan $J_1=0$ tenglik o'rinli.

(8) tenglikning 3-hadidagi integralda AD va BC da $dx=0$ bo'ladi. U holda quyidagi

$$J_2=\int_{\Gamma_\varepsilon} u_x u_{xy}dx=\int_{AB} u_x(x,0)u_{xy}(x,0)dx+\int_{CD} u_x(x,1)u_{xy}(x,1)dx$$

tenglik o’rinli. Bir jinsli shartga asosan $J_2=0$ kelib chiqadi.

AB va CD chiziqlarda $u_x u_{xx}$ dan olingan integralni hisoblaymiz, ularda $dy=0$. Bundan

$$J_3=\int_{\Gamma_\varepsilon} u_x u_{xx}dy=\int_{BC} u_x(1,y)u_{xx}(1,y)dy+\int_{DA} u_x(0,y)u_{xx}(0,y)dy$$

tenglik o’rinli. (4), (5) chegaraviy shartlarni hisobga olib,

$$\int_{BC} u_x(1,y)u_{xx}(1,y)dy+\int_{DA} u_x(0,y)u_{xx}(0,y)dy=\int_{BC} u_x(1,y)u_{xx}(1,y)dy$$

ko’rinishga kelamiz. Bu integralda $y=-x+1$ va $dy=-dx$ almashtirish bajaramiz, hamda teorema shartiga asosan $\psi_1^{'}(1)=\psi_0^{'}(1)=0$ ekanligidan,

$$\int_0^1 u_x(1,y)u_{xx}(1,y)dy=\int_0^1 u_x(1,-x+1)u_{xx}(1,-x+1)dx=\frac{1}{2}\int_0^1\left(u_x^2(1,1-x)\right)_x dx=$$

$$=\frac{1}{2}u_x^2(1,1-x)\Bigg|_{x=0}^{x=1}=\frac{1}{2}\left[u_x^2(1,0)-u_x^2(1,1)\right]=0$$

qiymat kelib chiqadi. U holda (8) ifoda $\varepsilon\to 0$ da

$$\iint_D (u_{xx}^2+u_{xy}^2)dxdy+\iint_D c(x,y)u^2dxdy=0 \qquad (9)$$

ko’rinishni oladi.

Oxirgi (9) tenglikda:

a) $c(x,y)>0$ bo’lsa, u holda $u\equiv 0$ ekani kelib chiqadi;

b) $c(x,y)=0$ bo’lsa, $u_{xy}=u_{xx}=0$ tengliklarni olamiz, bundan birini $u_{xx}=0$ tenglamani integrallab, $u=f(y)x+g(y)$ ga ega bo’lamiz. Bir jinsli chegaraviy shartlarga asosan $u\equiv 0$ ekani kelib chiqadi. **Teorema isbotlandi.**

5. Masala yechimining mavjudligi.

Teorema. Agar quyidagi

$$\varphi_i^{''}(y),(0\le y\le 1),\psi_i^{''}(x),i=0,1,$$

$$\psi_3(x),\psi_2(x),\varphi_3^{'}(y),\psi_2^{'}(x),\ f(x,y)$$

funksiyalar $0\le x\le 1,\quad 0\le y\le 1$ oraliqlarda uzluksiz bo‘lib,

$$\psi_1^{'}(1)=\psi_0^{'}(1)=0$$

bo‘lsa, masala yechimi mavjud.

Isbot. (1) tenglamada $u_{xx}=\upsilon$ belgilashni kiritamiz. U holda

$$\Delta\upsilon=f(x,y)-c(x,y)u(x,y) \qquad (10)$$

ko'rinishidagi tenglama hosil bo'ladi. Belgilashga asosan (2), (3) va (4) chegaraviy shartlari esa

$$\upsilon(0,y)=u_{xx}(0,y)=\overline{\varphi_3}(y)$$
$$\upsilon(1,y)=u_{xx}(1,y)=\overline{\varphi_1}(y)$$
$$\upsilon(x,0)=u_{xx}(x,0)=\psi_0''(x) \qquad (11)$$
$$\upsilon(x,1)=u_{xx}(x,1)=\psi_1''(x)$$

ko'rinishni oladi. Bu yerda $u_{xx}(0,y)=\overline{\varphi_3}(y)$ va $u_{xx}(1,y)=\overline{\varphi_1}(y)$- funksiyalar noma'lum.

(10) tenglamaning (11) chegaraviy shartlarni qanoatlantiruvchi yechimi quyidagicha bo'ladi:

$$\upsilon(x,y)=\int_0^1 G_\eta(x,y,\xi,0)\psi_0''(\xi)d\xi-\int_0^1 G_\eta(x,y,\xi,1)\psi_1''(\xi)d\xi+$$
$$+\int_0^1 G_\xi(x,y,0,\eta)\overline{\varphi_3}(\eta)d\eta-\int_0^1 G_\xi(x,y,1,\eta)\overline{\varphi_1}(\eta)d\eta+ \qquad (12)$$
$$+\iint_D G(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta-\iint_D G(x,y,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta$$

Bu yerda

$$G(x,y,\xi,\eta)=-\ln r+g(x,y,\xi,\eta), \quad \left(r=\sqrt{(x-\xi)^2+(y-\eta)^2}\right)$$

Laplas tenglamasi uchun to'rtburchak sohada Dirixle masalasining Grin funksiyasi.

$w=u_x$ belgilash kiritaylik, u holda quyidagi 2 ta masalaga ega bo'lamiz:

$$\begin{cases} w_x=\upsilon(x,y) \\ w(0,y)=\varphi_3(y) \end{cases} \qquad (13)$$

va

$$\begin{cases} u_x=w \\ u(x,-x+1)=\psi_2(x) \end{cases} \qquad (14)$$

(13) masalaning yechimi:

$$w=\int_0^x \upsilon(t,y)dt+\varphi_3(y) \qquad (15)$$

ko'rinishida bo'ladi. (14) masalami yechimini quyidagicha yozamiz:

$$u=\int_{1-y}^x w(t,y)dt+\psi_2(1-y) \qquad (16)$$

(12) va (15) ifodalar fordamida (16) formulani quyidagicha yozish mumkin:

$$u=\int_{1-y}^{x}\left[\int_{0}^{t}\left[\int_{0}^{1}G_{\eta}\left(t_1,y,\xi,0\right)\psi_0''\left(\xi\right)d\xi-\int_{0}^{1}G_{\eta}\left(t_1,y,\xi,1\right)\psi_1''\left(\xi\right)d\xi+\right.\right.$$

$$\left.\left.+\int_{0}^{1}G_{\xi}\left(t_1,y,0,\eta\right)\overline{\varphi_3}\left(\eta\right)d\eta-\int_{0}^{1}G_{\xi}\left(t_1,y,1,\eta\right)\overline{\varphi_1}\left(\eta\right)d\eta\right]dt_1\right]dt+\qquad(17)$$

$$+\int_{1-y}^{x}\left[\varphi_3\left(y\right)-\int_{0}^{t}\iint_{D}G\left(t_1,y,\xi,\eta\right)c\left(\xi,\eta\right)u\left(\xi,\eta\right)d\xi d\eta dt_1\right]dt+$$

$$+\int_{1-y}^{x}\int_{0}^{t}\left[\iint_{D}G\left(t_1,y,\xi,\eta\right)f\left(\xi,\eta\right)d\xi d\eta\right]dt_1dt+\psi_2(1-y)$$

(17) tenglikdagi ba’zi integrallarni hisoblaymiz:

$$J_0=\int_{1-y}^{x}\left(\int_{0}^{t}G\left(t_1,y,\xi,\eta\right)dt_1\right)dt=\int_{1-y}^{x}\int_{0}^{t}\left(-\ln\sqrt{\left(t_1-\xi\right)^2+\left(y-\eta\right)^2}+g\left(t_1,y,\xi,\eta\right)\right)dt_1dt=$$

$$=-\frac{1}{4}\left(\left(x-\xi\right)^2+\left(y-\eta\right)^2\right)\ln\left(\left(x-\xi\right)^2+\left(y-\eta\right)^2\right)+g_1(x,y,\xi,\eta)$$

$$J_1=\int_{1-y}^{x}\int_{0}^{1}\left[\int_{0}^{t}G_{\xi}\left(t_1,y,0,\eta\right)\overline{\varphi}_3\left(\eta\right)d\eta\right]dt_1dt=\int_{1-y}^{x}\left[\int_{0}^{1}\overline{\varphi}_3\left(\eta\right)d\eta\int_{0}^{t}G_{\xi}\left(t_1,y,0,\eta\right)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\varphi_3}\left(\eta\right)\left[x\ln\left[\frac{x^2+\left(y-\eta\right)^2}{\left(y-\eta\right)^2}\right]+\left(y-1\right)\ln\left[\frac{(1-y)^2+\left(y-\eta\right)^2}{\left(y-\eta\right)^2}\right]+g_2(x,y,0,\eta)\right]d\eta$$

$$J_2=\int_{1-y}^{x}\int_{0}^{1}\left[\int_{0}^{t}G_{\xi}\left(t_1,y,1,\eta\right)\overline{\varphi}_1\left(\eta\right)d\eta\right]dt_1dt=\int_{1-y}^{x}\left[\int_{0}^{1}\overline{\varphi}_1\left(\eta\right)d\eta\int_{0}^{t}G_{\xi}\left(t_1,y,1,\eta\right)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\varphi_1}\left(\eta\right)\left[\left(x-1\right)\ln\left[\frac{\left(x-1\right)^2+\left(y-\eta\right)^2}{1+\left(y-\eta\right)^2}\right]+y\ln\left[\frac{y^2+\left(y-\eta\right)^2}{1+\left(y-\eta\right)^2}\right]+g_3(x,y,1,\eta)\right]d\eta$$

bu yerda

$$g_1=-\frac{1}{2}\int_{1-y}^{x}\left[\xi\ln\left(\xi^2+\left(y-\eta\right)^2\right)-2t+2\left(y-\eta\right)arctg\frac{t-\xi}{y-\eta}-2\left(y-\eta\right)arctg\frac{-\xi}{y-\eta}\right]dt+$$

$$+\int_{1-y}^{x}\int_{0}^{t}g(t_1,y,\xi,\eta)dt_1dt$$

$$g_2 = 2(1-x-y) + 2(y-\eta)\left(arctg\frac{x}{y-\eta} - arctg\frac{1-y}{y-\eta}\right) +$$

$$\int_{1-y}^{x}\int_{0}^{t} g_\xi(t_1, y, 0, \eta)dt_1 dt$$

$$g_3 = 2(1-x-y) + 2(y-\eta)\left(arctg\frac{x-1}{y-\eta} - arctg\frac{-y}{y-\eta}\right) +$$

$$\int_{1-y}^{x}\int_{0}^{t} g_\xi(t_1, y, 1, \eta)dt_1 dt$$

Topilganlardan foydalanib, $u(x, y)$ ni soddalashtiramiz:

$$u(x,y) + \frac{1}{4}\iint_D R(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta = \overline{F}(x,y) \qquad (18)$$

Bu yerda

$$\overline{F}(x,y) = F_1(x,y) + F_2(x,y)$$

$$F_1(x,y) = \frac{1}{4}\iint_D \overline{R}(x,y,\xi,\eta) f(\xi,\eta)d\xi d\eta$$

$$F_2(x,y) = \int_0^1\left[\int_{1-y}^{x}\int_0^t G_\eta(t_1,y,\xi,0)dt_1 dt\right]\psi_0''(\xi)d\xi - \int_0^1\left[\int_{1-y}^{x}\int_0^t G_\eta(t_1,y,\xi,1)dt_1 dt\right]\psi_1''(\xi)d\xi +$$

$$+\int_0^1\overline{\varphi_3}(\eta)\left[x\ln\left[\frac{x^2+(y-\eta)^2}{(y-\eta)^2}\right] + (y-1)\ln\left[\frac{(y-1)^2+(y-\eta)^2}{(y-\eta)^2}\right] + g_2(x,y,0,\eta)\right]d\eta -$$

$$-\int_0^1\overline{\varphi_1}(\eta)\left[(x-1)\ln\left[\frac{(x-1)^2+(y-\eta)^2}{1+(y-\eta)^2}\right] + y\ln\left[\frac{y^2+(y-\eta)^2}{1+(y-\eta)^2}\right] + g_3(x,y,1,\eta)\right]d\eta +$$

$$+(x+y-1)\varphi_3(y) + \psi_2(1-y);$$

$$R(x,y,\xi,\eta) = \overline{R}(x,y,\xi,\eta)c(\xi,\eta)$$

$$\overline{R}(x,y,\xi,\eta) = \left((x-\xi)^2+(y-\eta)^2\right)\ln\left((x-\xi)^2+(y-\eta)^2\right) + g_1(x,y,\xi,\eta)$$

(18) – Fredgolmning 2-tur integral tenglamasi bo‘lib, ko‘rish mumkinki, uning yadrosi $R(x,y,\xi,\eta)$ - uzluksiz funksiya, $\overline{F}(x,y)$ funksiya ham chegaraviy shartlardagi berilgan funksiyalarga qo‘yilgan shartlarga asosan uzluksiz bo‘ladi. Shuning uchun, unga Fredgolm teoremalarini qo‘llash mumkin. Shunday qilib, uning yechimi[1]:

$$u(x,y)=\overline{F}(x,y)-\frac{1}{4}\iint_D \Gamma(x,y,\xi,\eta)\overline{F}(\xi,\eta)d\xi d\eta \qquad (19)$$

ko’rinishida bo’ladi. Bu yerda $\Gamma(x,y,\xi,\eta)-R(x,y,\xi,\eta)$ yadroning rezolventasi. $\overline{F}(x,y)$ ni ifodasidan foydalanib, (19) ni quyidagicha yozamiz:

$$u(x,y)=\int_0^1\overline{\varphi_3}(\eta)\left[x\ln\left[\frac{x^2+(y-\eta)^2}{(y-\eta)^2}\right]+(y-1)\ln\left[\frac{(1-y)^2+(y-\eta)^2}{(y-\eta)^2}\right]+g_2(x,y,0,\eta)\right]d\eta-$$

$$-\int_0^1\overline{\varphi_1}(\eta)\left[(x-1)\ln\left[\frac{(x-1)^2+(y-\eta)^2}{1+(y-\eta)^2}\right]+y\ln\left[\frac{y^2+(y-\eta)^2}{1+(y-\eta)^2}\right]+g_3(x,y,1,\eta)\right]d\eta-$$

$$-\frac{1}{4}\iint_D\Gamma(x,y,\xi,\eta)\left[\int_0^1\overline{\varphi_3}(\eta_1)\left[\xi\ln\left[\frac{\xi^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]+\right.\right. \qquad (20)$$

$$\left.\left.+(\eta-1)\ln\left[\frac{(1-\eta)^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]+g_2(\xi,\eta,0,\eta_1)\right]d\eta_1 d\xi d\eta+\frac{1}{4}\iint_D\Gamma(x,y,\xi,\eta)\left[\int_0^1\overline{\varphi_1}(\eta_1)\left[(\xi-1)\ln\left[\frac{(\xi-1)^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]+\right.\right.$$

$$\left.\left.+\eta\ln\left[\frac{\eta^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]+g_3(\xi,\eta,1,\eta_1)\right]\right]d\eta_1 d\xi d\eta+F_3(x,y)$$

Bu yerda

$$F_3(x,y)=\frac{1}{4}\iint_D\overline{R}(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta+\int_0^1\left[\int_{1-y}^x\int_0^t G_\eta(t_1,y,\xi,0)dt_1dt\right]\psi_0''(\xi)d\xi-$$

$$-\int_0^1\left[\int_{1-y}^x\int_0^t G_\eta(t_1,y,\xi,1)dt_1dt\right]\psi_1''(\xi)d\xi+(x+y-1)\varphi_3(y)+\psi_2(1-y)$$

$u_{xx}(0,y)=\overline{\varphi_3}(y)$ ekanligidan foydalanib, (20) dan x bo‘yicha ikki marta hosila olib, $x=0$ qo‘ysak, Fredgolmning 2-tur integral tenglamasiga kelamiz:

$$\overline{\varphi_3}(y)=\int_0^1 R_1(y,\eta_1)\overline{\varphi_3}(\eta_1)d\eta_1+\int_0^1 k_1(y,\eta_1)\overline{\varphi_1}(\eta_1)d\eta_1+F_3(y) \qquad (21)$$

Bu yerda

$$R_1(y,\eta_1)=\iint_D\Gamma_{xx}(0,y,1,\eta_1)\left[\xi\ln\left[\frac{\xi^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]+(\eta_1-1)\ln\left[\frac{(1-\eta)^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]+g_{2xx}(\xi,\eta,1,\eta_1)\right]d\xi d\eta$$

$$k_1(y,\eta_1)=\frac{2}{1+(y-\eta_1)^2}+\frac{4(y-\eta_1)^2}{\left(1+(y-\eta_1)^2\right)^2}+g_{3xx}(0,y,1,\eta_1)+$$

$$+\frac{1}{4}\iint_D \Gamma_{xx}(0,y,\xi,\eta)\left[(\xi-1)\ln\left[\frac{(\xi-1)^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]+\eta\ln\left[\frac{\eta^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]\right]d\xi d\eta$$

$$F_3(y)=F_{3xx}(0,y)$$

Endi $u_{xx}(1,y)=\overline{\varphi_1}(y)$ ekanligidan foydalanib, (20) da x bo‘yicha ikki marta hosila olamiz va $x=1$ qo‘ysak, Fredgolmning 2-tur integral tenglamasiga kelamiz:

$$\overline{\varphi_1}(y)=\int_0^1 R_2(y,\eta_1)\overline{\varphi_1}(\eta_1)d\eta_1+\int_0^1 k_2(y,\eta_1)\overline{\varphi_3}(\eta_1)d\eta_1+\overline{F_3}(y) \qquad (22)$$

Bu yerda

$$R_2(y,\eta_1)=\iint_D \Gamma_{xx}(1,y,1,\eta_1)\left[(\xi-1)\ln\left[\frac{(\xi-1)^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]+\eta\ln\left[\frac{\eta^2+(\eta-\eta_1)^2}{1+(\eta-\eta_1)^2}\right]+g_{3xx}(x,y,1,\eta_1)\right]d\xi d\eta$$

$$k_1(y,\eta_1)=\frac{2}{1+(y-\eta_1)^2}+\frac{4(y-\eta_1)^2}{\left(1+(y-\eta_1)^2\right)^2}+g_{2xx}(1,y,0,\eta)-$$

$$-\frac{1}{4}\iint_D \Gamma_{xx}(1,y,\xi,\eta)\left[\xi\ln\left[\frac{\xi^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]+(\eta_1-1)\ln\left[\frac{(1-\eta)^2+(\eta-\eta_1)^2}{(\eta-\eta_1)^2}\right]\right]d\xi d\eta$$

$$F_4(y)=g_{2xx}(1,y,1,\eta)+\iint_D \Gamma_{xx}(1,y,\xi,\eta)\int_0^1 g_2(x,y,1,\eta_1)d\eta_1 d\xi d\eta+$$

$$+g_{3xx}(1,y,0,\eta)+\iint_D \Gamma_{xx}(1,y,\xi,\eta)\int_0^1 g_3(x,y,0,\eta_1)d\eta_1 d\xi d\eta+F_3(y)$$

$$\overline{F_3}(y)=F_{3xx}(1,y)$$

Shunday qilib, noma‘lum funksiyalarga nisbatan Fredgolmning 2-tur integral tenglamalarini hosil qilamiz.

(21) va (22) integral tenglamalardan $\overline{\varphi_1}(y)$, $\overline{\varphi_3}(y)$ noma‘lumlarni topib, (20) ifodaga olib borib, berilgan masala yechimini topamiz.

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