# GEOMETRIYA KURSIDA VEKTORLAR NAZARIYASINING AMALIY QO‘LLANILISHI

**Abdurashidov Nuriddin**
**Xolboyev Sanjar**
**Eshtemirov Eshtemir**
**Jo‘rayeva Sevinch**
*Denov tadbirkorlik va pedagogika instituti Oliy matematika kafedrasi o’qituvchi*
*abdurashidovnuriddin9550@gmail.com*
*xolboyevsanjar0911@mail.ru*
*eshtemireshtemirov577@gmail.com*
*jo‘rayevasevinch179@gmail.com*

**Anatatsiya:** *Ushbu maqolada vektorlarning skalyar, vektor va aralash ko’paytmalari yordamida geometriya masalalarini yechishni ko’rib o’tamiz.*

**Kalit so’zlar:** *Vektorlar ko’paytma, skalyar ko’paytma, aralash ko’paytma, yo’naltiruvchi vektor, komplanarlik.*

Ta’rif. $\overline{a},\ \overline{b}$ vektorlarning uzunliklari bilan ular orasidagi burchak kosinusini ko’paytirishdan hosil qilingan son bu vektorlarning skalyar ko’paytmasi deb ataladi.

$\overline{a},\ \overline{b}$ vektorlarning skalyar ko’paytmasi $\overline{a}\ \overline{b}$ yoki $(\overline{a}\ \overline{b})$ ko’rinishda belgilanadi.

Demak, $\overline{a}\ \overline{b} = |\overline{a}||\overline{b}|\cos\varphi$, $\varphi = (\bar{a} \wedge \vec{b})$.

$\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorning aralash ko’paytmasi deb, (vektorlarning ko’rsatilgan tartibiga ko’ra) $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko’paytmasidan iborat vektorni $\vec{c}$ vektorga skalyar ko’paytirishdan hosil qilingan songa aytiladi:

$([\vec{a}, \vec{b}], \vec{c})$

Aralash ko’paytma

$(\vec{a}, \vec{b}, \vec{c}) = ([\vec{a}, \vec{b}], \vec{c})$

ko’rinishda belgilanadi. Aralash ko’paytmaning geometrik ma’nosi quyidagicha: $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar biror O nuqtaga qo’yilgan bo’lib, komplanar bo’lmasin hamda o’ng uchlikni hosil qilsin. Qirralari shu berilgan vektorlardan iborat parallelepipedni yasasak, $|[\vec{a}, \vec{b}]|$ miqdor shu parallelepiped asosining yuzini bildiradi. Aralash ko’paytma ta’rifiga asosan $([\vec{a}, \vec{b}], \vec{c}) = |[\vec{a}, \vec{b}]||\vec{c}|\cos\varphi$, bu yerda φ, $[\vec{a}, \vec{b}]$ va $\vec{c}$ vektorlar orasidagi burchak bo’lib, $|\vec{c}|\cos\varphi$ miqdor $\vec{c}$ vektorning $[\vec{a}, \vec{b}]$ vektor yo’nalishidagi to’g’ri chiziqdagi proyeksiyasiga teng bo’lib, parallelepipedning balandligidir. ($|\vec{c}|\cos\varphi = h$).

Demak, agar $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar o'ng uchlik hosil qilsa, bu vektorlarning aralash ko'paytmasi bu vektorlarga yasalgan parallelepiped hajmiga teng bo'ladi. Agar $\vec{a}$, $\vec{b}$, $\vec{c}$ lar chap uchlik tashkil qilsa, $[\vec{a},\vec{b}]$ vektor bilan $\vec{c}$ orasidagi burchak $\varphi \geq \frac{\pi}{2}$ $(\cos\varphi \leq 0)$ bo'ladi. U holda $([\vec{a},\vec{b}],\vec{c}) = -V$. Agar $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar o'zlarining koordinatalari

$\vec{a}=\{X_1;\ Y_1;\ Z_1\}$, $\vec{b}=\{X_2;\ Y_2;\ Z_2\}$, $\vec{c}=\{X_3;\ Y_3;\ Z_3\}$ bilan berilgan bo'lsa, u holda aralash ko'paytma.

$$([\vec{a},\vec{b}],\vec{c}) = (\vec{a},\vec{b},\vec{c}) = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix}$$

$1^0$. $\left(\vec{a}\,\vec{b}\,\vec{c}\right)=\left(\vec{b}\,\vec{c}\,\vec{a}\right)$. Haqiqatan ham, bu uch vektorga qurilgan parallelepiped hajmlarining absolyut qiymatlari teng, undan tashqari $\vec{a}, \vec{b}, \vec{c}$ uchlik bilan $\vec{b}, \vec{c}, \vec{a}$ uchlikning orientatsiyalari bir xil. Shuning singari $\left(\vec{a}\,\vec{b}\ \vec{c}\right)=\left(\vec{b}\,\vec{c}\,\vec{a}\right)=\left(\vec{c}\,\vec{a}\,\vec{b}\right)$.

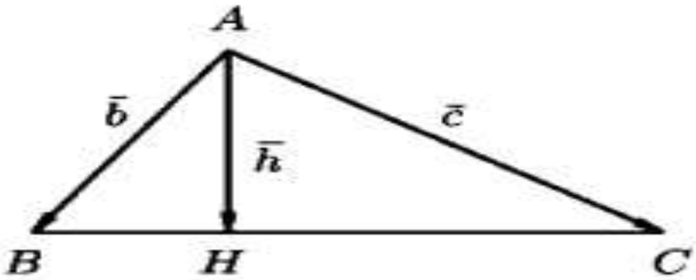
$2^0$. $\left(\vec{a}\ \vec{b}\,\vec{c}\right)=-\left(\vec{b}\ \vec{a}\,\vec{c}\right)$, chunki $\left(\vec{a}\ \vec{b}\,\vec{c}\right)=[\vec{a}\ \vec{b}]\vec{c}=-[\vec{b}\ \vec{a}]\vec{c}=-\left(\vec{b}\ \vec{a}\,\vec{c}\right)$. demak, $\left(\vec{a}\,\vec{b}\,\vec{c}\right)=-\left(\vec{b}\,\vec{a}\,\vec{c}\right)$, $\left(\vec{b}\,\vec{c}\,\vec{a}\right)=-\left(\vec{c}\,\vec{b}\,\vec{a}\right)$, $\left(\vec{c}\,\vec{a}\,\vec{b}\right)=\left(\vec{a}\,\vec{c}\,\vec{b}\right)$.

$3^0$. $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$ chunki $((\vec{a}+\vec{b})\vec{c}\,\vec{d})=[\vec{a}+\vec{b}\quad \vec{c}]\vec{d}=$
$=([\vec{a}\ \vec{c}]+[\vec{b}\ \vec{c}]\vec{d})=[\vec{a}\ \vec{c}]\vec{d}+[\vec{b}\ \vec{c}]\vec{d}=(\vec{a}\,\vec{c}\,\vec{d})+(\vec{b}\,\vec{c}\,\vec{d})$.

$4^0$. $\forall\lambda\in R$ uchun $(\lambda\vec{a}\,\vec{b}\,\vec{c})=\lambda(\vec{a}\,\vec{b}\,\vec{c})$, chunki $(\lambda\vec{a}\,\vec{b}\,\vec{c})=[\lambda\vec{a}\,\vec{b}\,]\vec{c}=$
$=\lambda[\vec{a}\ \vec{b}]\vec{c}=\lambda(\vec{a}\,\vec{b}\,\vec{c})$.

$5^0$. $\vec{a}, \vec{b}, \vec{c}$ komplanar bo'lsa, ularning aralash ko'paytmasi nolga teng, chunki ularga qurilgan parallelepiped tekislikda joylashib qoladi, bunday parallelepipedning balandligi nolga tengligidan hajmi ham nolga teng; aksincha $(\vec{a}\,\vec{b}\,\vec{c})=0 \Rightarrow \vec{a}, \vec{b}, \vec{c}$ vektorlar komplanar. Haqiqatan ham $(\vec{a}\,\vec{b}\,\vec{c})=0 \Rightarrow [\vec{a}\ \vec{b}]\vec{c}=0$ yoki $[\vec{a}\ \vec{b}]\perp\vec{c}$. Lekin vektor ko'paytmaning ta'rifiga asosan $[\vec{a}\ \vec{b}]\perp\vec{a}$, $[\vec{a}\ \vec{b}]\perp\vec{b}$, bundan $[\vec{a}\ \vec{b}]$ vektorning $\vec{a}, \vec{b}, \vec{c}$ ning har biriga perpendikularligi kelib chiqadi, demak, $\vec{a}, \vec{b}, \vec{c}$ komplanar.

1-misol: ABC uchburchakda $\overrightarrow{AB}=\vec{b}$, $\overrightarrow{AC}=\vec{c}$. $\overrightarrow{AH}$ balandlik bo'yicha yo'nalgan $\vec{h}$ vektorni $\vec{a}$ va $\vec{c}$ lar orqali ifodalang.




$\vec{a}$

$\overline{h} = \overline{c} - \overline{Hc}$ ga egamiz. Bu yerda $\overline{Bc} = \overline{Hc}$ bo’ladi. $\overline{Hc} = \alpha \cdot \overline{Bc}$ deb belgilab olamiz.

$\overline{Bc}$ esa $\overline{a}$ vektorga teng deb olsak, $\overline{Hc} = \alpha \cdot \overline{c}$ bo’ladi. $\overline{hc} = \alpha \cdot \overline{Bc}$ bo’ladi.

$\overline{AH} \perp \overline{BC}$ bundan esa $\overline{AH} \cdot \overline{BC} = 0$ ga teng bo’ladi.

Bizda ma’lumki $\overline{Hc} = \alpha \cdot \overline{a}$ ga $\overline{Bc}$ esa $\overline{a}$ ga teng . Bular orqali $\overline{AH}$ ni topib olamiz. $\overline{AH} = \overline{c} - \alpha \cdot \overline{a}$ endi $(\overline{c} - \alpha \cdot \overline{a}) \cdot$ a=0 $\overline{c} \cdot \overline{a} - \alpha \cdot \overrightarrow{a^2} = 0$ α ni olib borib topamiz $\overline{h} = \overline{c} - \frac{\overline{c} \cdot \overline{a}}{|a|^2} \cdot \overline{a}$

2-misol $\vec{a}, \vec{b}, \vec{c}$ - ixtiyoriy vektorlar va $\alpha, \beta, \gamma$ - ixtiyoriy haqiqiy sonlar bo’lsa, $\alpha\vec{a} - \beta\vec{b},\ \gamma\vec{b} - \alpha\vec{c},\ \beta\vec{c} - \gamma\vec{a}$ vektorlarning komplanar ekanligi isbotlansin.

Isbot. Shu uchta vektorning $(\alpha\vec{a} - \beta\vec{b}\ \ \gamma\vec{b} - \alpha\vec{c}\ \ \beta\vec{c} - \gamma\vec{a})$ aralash ko’paytmasini hisoblaylik. Buning uchun yuqorida keltirilgan $1^0$ – $5^0$-xossalarni nazarda tutsak,

$$(\alpha\vec{a} - \beta\vec{b}\ \gamma\vec{b} - \alpha\vec{c}\ \ \beta\vec{c} - \gamma\vec{a}) = (\alpha\vec{a}(\gamma\vec{b} - \alpha\vec{c})(\beta\vec{c} - \gamma\vec{a}) -$$

$$-(\beta\vec{b}(\gamma\vec{b} - \alpha\vec{c})(\beta\vec{c} - \gamma\vec{a})) = (\alpha\vec{a}\,\gamma\vec{b}\ \beta\vec{c} - \gamma\vec{a}) - (\alpha\vec{a}\ \beta\vec{c}\alpha\vec{c} - \gamma\vec{a}) -$$

$$-(\beta\vec{b}\,\gamma\vec{b}\ \beta\vec{c} - \gamma\vec{a}) + (\vec{\beta}\vec{b}\alpha\vec{c}\beta\vec{c} - \gamma\vec{a}) = \alpha\gamma\beta(\vec{a}\vec{b}\vec{c}) - \alpha\gamma\gamma(\vec{a}\vec{b}\vec{a}) - \alpha\alpha\beta(\vec{a}\vec{c}$$

$$+\alpha\alpha\gamma(\vec{a}\vec{c}\vec{a}) - \beta\gamma\beta(\vec{b}\vec{b}\vec{c}) + \beta\gamma\beta(\vec{b}\vec{b}\vec{c}) + \beta\gamma\gamma(\vec{b}\vec{b}\vec{a}) + \beta\alpha\beta(\vec{b}\vec{c}\vec{c}) -$$

$$-\beta\alpha\gamma(\vec{b}\vec{c}\vec{a}) = \alpha\beta\gamma(\vec{a}\vec{b}\vec{c}) - \alpha\beta\gamma(\vec{a}\vec{b}\vec{c}) = 0.$$

Aralash ko’paytmasi nolga teng bo’lgani uchun $5^0$ ga asosan ular komplanardir.

3-misol . $\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2) vektorlarga qurilgan tetraedr

berilgan . Quyidagilar topilsin : a) tetraedrning hajmi , b) ABC yoqning yuzi , v) D uchdan

tushurilgan bandlik , g) AB va BC qirralar orasidagi $\varphi_1$ burchak kosinusi , d) ABC va ADC yoqlar orasidagi $\varphi_2$ burchak kosinusi. $\overrightarrow{AB}$ (2,0.0), $\overrightarrow{AC}$ (3,4,0), $\overrightarrow{AD}$ (3,4,2)

a) Tetraedr hajmi .

b) ABC yoqning yuzi .

c) D uchidan tushirilgan balandlik .

a) Formulaga kora $V = \pm\frac{1}{6}\overrightarrow{AB}*\overrightarrow{AC}*\overrightarrow{AD} = \pm\frac{1}{6}\begin{vmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 3 & 4 & 2 \end{vmatrix} =$

$= \pm\frac{1}{6}(16 + 0 + 0 - 0 - 0) = \pm\frac{1}{6} * 16 = \frac{8}{3}$ kub birlik

Maktab giometryasidan ma’lumki $V = \frac{1}{3} S_a {*}h \Rightarrow h = \frac{3*V}{S_{ACB}}$

b) $S_{ACB} = \frac{1}{2} \left|\overrightarrow{AC} \times \overrightarrow{AB}\right|$

$$\overrightarrow{AC} \times \overrightarrow{AB} = \begin{vmatrix} i & j & k \\ 3 & 4 & 0 \\ 2 & 0 & 0 \end{vmatrix} = (0 + 0 + 0 - 8k - 0 - 0) = 8k$$

$$S_{ACB} = \frac{1}{2} * 8 = 4$$

v) $h = \frac{3*V}{S_{ACB}} = \frac{\frac{8}{3}*3}{4} = \frac{8}{4} = 2$ $\quad h = 2$ .

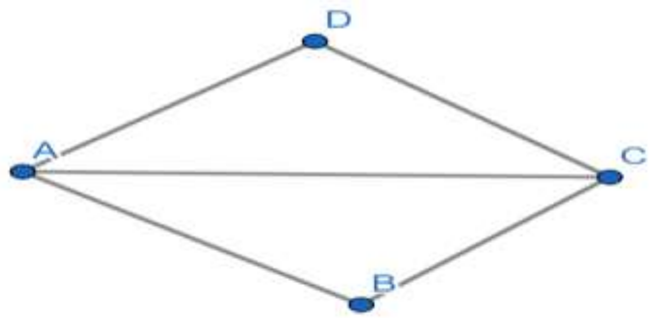
2 ) $\overrightarrow{AB} = \overline{BA}$ $\quad \overrightarrow{AC} - \overrightarrow{AB} = \bar{C} - \bar{B} = \overrightarrow{BC}$

$\overrightarrow{AC} = \bar{C} - \bar{A}_,$ $\quad \overrightarrow{AB}$ va $\overrightarrow{BC}$ qirralar orasidagi burchak

$\cos\varphi = \frac{\bar{a}\bar{b}}{|a||b|}$ $\quad \overrightarrow{AB}$ (2,0.0) BC(1,4,0)

$\cos\varphi = \frac{\overline{AB}*\overline{BC}}{|AB||BC|} = \frac{2+0+0}{\sqrt{4}*\sqrt{1+16}} = \frac{2}{2*\sqrt{17}} = \frac{1}{\sqrt{17}}$

ABC va ADC yoqlar orasidagi $\varphi$ burchak kosinusi




$\varphi_2 -?$

$1^0$ Buning uchun $\Delta ABC$ dan $\overrightarrow{AB}$ va $\overrightarrow{AC}$ yo’naltiruvchisini va $\Delta ADC$ dan esa $\overrightarrow{AD}$ va $\overrightarrow{AC}$ ni yo’naltiruvchisini topami.

$\overrightarrow{AB}$ va $\overrightarrow{AC}$ to’g’ri chiziq tenglamasiga ko’ra

$\frac{x-2}{3-2} = \frac{y-4}{4-0} = \frac{z-0}{0-0}$ $\quad \frac{x-2}{1} = \frac{y}{4} = \frac{z}{0}$ $\quad$ yo’naltiruvchisi $\vec{n}(1;4;0)$

$2^0$ $\overrightarrow{AD}$ va $\overrightarrow{AC}$ dan esa:

$\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2-0}$ $\quad \frac{x-3}{0} = \frac{y-4}{0} = \frac{z-2}{2}$ $\quad$ yo’naltiruvchisi esa $\vec{l}(0;0;2)$

$\cos(n \wedge l) = \frac{0}{\sqrt{17}} * \frac{1}{\sqrt{4}} = 0$ $\quad \varphi = 90^0$

**FOYDALANILGAN ADABIYOTLAR:**

1. N.D.Dodajonov, M.SH.Jo’rayeva, “Geometriya” 1-qism,Toshkent <<O’qituvchi>> 1996.

2. S.V. B A X V A L O V , P .S .M O D E N O V , A .S .P A R X O M E N K O, Analitik geometriyadan masalalar to’plami Toshkent-2005 546-bet.

3. B.A.Xudayarov.”Chiziqli algebra va analitik geometriya” Toshkent-2018, 284 bet.

4. Sh.A.Ayupov, B.A.Omirov,A.X.Xudoyberdiyev, F.H.Haydarov. Algebra va sonlar nazariyasi. Toshkent-2019 (62-67)

5. R.N.Nazarov, B.T.Toshpo’latov, A.D.Do’sumbetov. Algebra va sonlar nazariyasi. I qism. Toshkent-1993.(216-219)

6. Malik D.S., Mordeson J.N., Sen M.K., Fundamentals of abstract algebra(o’quv qo’llanma) WCB McGrew-Hill, 1997, p.636.

7. Allakov. Sonlar nazariyasidan misol va masalalar. (o’quv qo’llanma)

– T. Inovatsion rivojlanish nashriyot matbaa uyi. 2020.348b.

8. Abdurashidov N.G’.”Simmetrik Li va Leybnits algebralari va ularning xossalari”. “O’ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR” APREL 2022. (7), 62-63.

9. Eshtemirov Eshtemir Salim o‘g’li, Abdurashidov Nuriddin G’iyoziddin o‘g’li. VEYL-TITCHMARSH FUNKSIYASI VA SPEKTRAL FUNKSIYA ORASIDAGI MUNOSABAT. “O’ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR” 20-iyun 2023-yil 20-son (870-875).

10. Abdurashidov N. G’., Eshtemirov E. S .SIMMETRIK LEYBNITS ALGEBRALARI VA ULARNING XOSSALARI. << Matematik modellashtirish va axborot texnologiyalarining dolzarb masalalari>> xalqaro ilmiy-amaliy anjuman. Nukus 2-3-may 2023-yil 1-Tom.

11. Abdurashidov Nuriddin , Toshtemirova Sarvara,Yo’ldoshev Husan.

Laplas teoremasi yordamida 4-tartibli determinantni hisoblash. “O’ZBEKISTONDA FANLARARO INNOVATSIYALAR VA ILMIY TADQIQOTLAR” 20-fevral 2024-yil 27-son (163-166)

12. Abdurashidov Nuriddin, Tog‟ayev Turdimurod, Rustamov Bilol, Eshtemirov Eshtemir “Equation of the Result of Second-Order Surfaces” excellencia: international multi-disciplinary journal of education https://multijournals.org/index.php/excellencia-imje

13. 5.Abdurashidov Nuriddin,Tog‘ayev Turdimurod,Rustamov Bilol. “Laplas tenglamasining fundamental yechimi” . “So‟ngi ilmiy tadqiqotlar nazariyasi” Ilmiy-uslubiy jurnali. 13-iyun 2024- yil 7-jild 6-son (33-37).

14. B.M.Rustamov, N.G‘.Abdurashidov, Sh.Ashirov, A.Saitniyozov. “” International Journal of Education, Social Science & Humanities. Finland Academic Research Science Publishers 369-372-bet 22-02-2025.

15. Abdurashidov Nuriddin, Tog’ayev Turdimurod, Rustamov Bilol, Eshtemirov Eshtemir “Equation of the Result of Second-Order Surfaces” EXCELLENCIA: INTERNATIONAL MULTI-DISCIPLINARY JOURNAL OF EDUCATION https://multijournals.org/index.php/excellencia-imje

16. Abdurashidov Nuriddin G'iyoziddin o'g'li ,Tog'ayev Turdimurod Xurram o'g'li,Rustamov Bilol Muxbiddinovich. "Laplas tenglamasining fundamental yechimi" . "So'ngi ilmiy tadqiqotlar nazariyasi" Ilmiy-uslubiy jurnali. 13-iyun 2024-yil 7-jild 6-son (33-37).

17. Rustamov Bilol, Abdurashidov Nuriddin, Eshtemirov Eshtemir, Baltabayeva Saida. "DERIVATION OF THE DISTANCE BETWEEN PARALLEL PLANES" Journal of Multidisciplinary Sciences and Innovations. German internatsional journals.company. 392-396- bett. 18.04.2025.