16. Abdurashidov Nuriddin G‘iyoziddin o‘g‘li ,Tog‘ayev Turdimurod Xurram o‘g‘li,Rustamov Bilol Muxbiddinovich. “Laplas tenglamasining fundamental yechimi” . “So’ngi ilmiy tadqiqotlar nazariyasi” Ilmiy-uslubiy jurnali. 13-iyun 2024-yil 7-jild 6-son (33-37).

17. Rustamov Bilol, Abdurashidov Nuriddin, Eshtemirov Eshtemir, Baltabayeva Saida. “DERIVATION OF THE DISTANCE BETWEEN PARALLEL PLANES” Journal of Multidisciplinary Sciences and Innovations. German internatsional journals.company. 392-396- bett. 18.04.2025.

OLIMPIADA MASALALAR VA ULARNING YECHISH USULLARI

Qurbonaliyeva Shoira Maxmanazarovna
SamDCHTI akademik litseyi matematika fani bosh o‘qituvchisi

Annotatsiya: *Mazkur maqolada matematika fanidan olimpiada masalalarining o‘ziga xos xususiyatlari, ularni yechish metodlari hamda o‘quvchilarning mantiqiy va ijodiy fikrlashini rivojlantirishdagi ahamiyati yoritilgan. Algebra, geometriya va sonlar nazariyasiga oid namunaviy masalalar hamda ularning yechimlari keltirilgan.*

Kalit so‘zlar: *matematika, olimpiada, masala, algebra, geometriya, sonlar nazariyasi, mantiqiy fikrlash.*

KIRISH

Matematika fanidan olimpiadalar iqtidorli o‘quvchilarni aniqlash va ularning matematik tafakkurini rivojlantirishda muhim vositadir. Olimpiada masalalari nostandart yondashuvni talab qiladi.

Olimpiada masalalarining xususiyatlari

- nostandart yechim;
- mantiqiy tahlil;
- isbotlash;
- ijodiy fikrlashni rivojlantirish.

Yechish metodlari

Matematik induksiya, invariantlar usuli, qarama-qarshilik usuli, ekstremal element usuli, tengsizliklardan foydalanish va kombinatorik tahlil.

1-masala.

Agar $x+\frac{1}{x}=3$ bo‘lsa, $x^2+\frac{1}{x^2}$ ni toping.

Yechim: $\left(x+\frac{1}{x}\right)^2=x^2+2+\frac{1}{x^2}=9$, bundan $x^2+\frac{1}{x^2}=7$.

Javob: 7.

2-masala

Har qanday ketma-ket uchta natural sonning ko‘paytmasi 6 ga bo‘linishini isbotlang.

Yechim: Uchta ketma-ket son orasida bittasi 3 ga, kamida bittasi 2 ga bo‘linadi. Demak, ko‘paytma 6 ga bo‘linadi.

3-masala. $abc \leq 1$ shartni qanoatlantiradigan musbat haqiqiy a ,b, c sonlar uchun $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq 1+\frac{6}{a+b+c}$ tengsizlikni isbotlang.

1- yechim: Tengsizlikni ikkala tomonini musbat $a+b+c$ ga ko‘paytirib olamiz , natijada tengsizlik $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c) \geq a+b+c+6$ ko‘rinishni oladi. Koshi tengsizligiga ko‘ra $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{3}{\sqrt[3]{abc}} \geq 3$ va $+b+c \geq 3\sqrt[3]{abc}$. Demak,

$\frac{1}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c)\geq(a+b+c)$ va

$\frac{2}{3}\cdot\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c)\geq\frac{2}{3}\cdot\frac{3}{\sqrt[3]{abc}}\cdot 3\sqrt[3]{abc}=6.$

Bu ikkala tengsizliklarni qo‘shamiz:

$$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c)\geq(a+b+c)+6$$

yoki $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 1+\frac{6}{a+b+c}$. Tengsizlik isbotlandi.

2-yechim: Tengsizlikni isbotlashda quyidagi ikki holni qaraymiz:

1-hol. $a+b+c\geq 3$, 2-hol. $a+b+c<3$.

1-holda $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{3}{\sqrt[3]{abc}}\geq 3\ \Rightarrow$

$\Rightarrow\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c)\geq 3\cdot(a+b+c)\geq a+b+c+6.$

2-holda $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot(a+b+c)\geq\frac{3}{\sqrt[3]{abc}}\cdot 3\sqrt[3]{abc}=9>\ a+b+c+6.$

Demak , har ikki holda ham $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 1+\frac{6}{a+b+c}$. Tengsizlik isbotlanadi.

4-masala. ABC uchburchakda A va C ichki burchaklarning bissektrisalari BC va AB tomonlari mos ravishda A_1 va C_1 nuqtalarda , ABC uchburchakka tashqi chizilgan aylana bilan mos ravishda A_2 va C_2 nuqtalarda kesishsin. K nuqta $A_1\,C_2$

va $C_1\,A_2$ to‘g‘ri chiziqlarning kesishish nuqtasi, I nuqta ABC uchburchakka ichki chizilgan aylana markazi bo‘lsin. KI to‘g‘ri chiziq AC tomonning o‘rtasidan o‘tishini isbotlang.

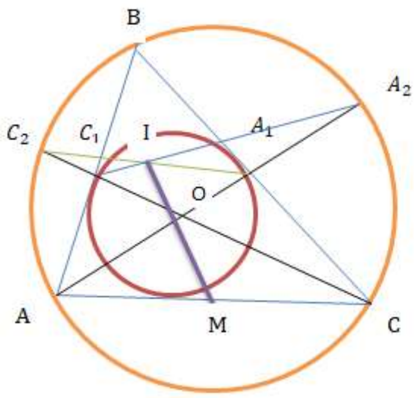
Yechim:

Masala shartiga ko‘ra

$\angle C_2AB=\angle C_2CB=\angle C_2CA=\angle C_2A_2\,A=\gamma$

$\angle A_2CB=\angle A_2AB=\angle A_2AC=\angle A_2\,C_2C=\alpha$

va $\angle AC_2C=\angle AA_2C=\angle ABC=2\beta$ deb olamiz. KI to‘g‘ri chiziq AC tomonni M nuqtada kessin. $\Delta\,C_2\,AC_1$ va $\Delta\,C_2\,AI$ lar uchun sinuslar teoremasiga ko‘ra :



$\frac{C_2\,C_1}{C_1I}=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin C_2\,IA}{\sin AC_2\,I}\Rightarrow\frac{C_2\,C_1}{C_1I}=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin(\alpha+\gamma)}{\sin 2\beta}.$

Xuddi shunday , $\Delta\,IA_2\,C$ uchun : $\frac{IA_1}{A_1A_2}=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin IA_2\,C}{\sin A_2\,IC}=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin 2\beta}{\sin(\alpha+\gamma)}.$

Demak , $\frac{C_2\,C_1}{C_1I}\cdot\frac{IA_1}{A_1A_2}=\left(\frac{\sin\gamma}{\sin\alpha}\right)^2$ (1)

$\Delta\,C_2\,A_2I$ uchburchak uchun Cheva teoremasiga ko‘ra

$\frac{C_2\,N}{NA_2}=\frac{C_2\,C_1}{C_1I}\cdot\frac{IA_1}{A_1A_2}=\left(\frac{\sin\gamma}{\sin\alpha}\right)^2$ (2)

U holda $\Delta\,C_2\,IN$ va $\Delta\,A_2IN$ lar uchun sinuslar teoremasiga va (2) ga ko‘ra

$$\frac{\sin C_2 IN}{\sin NIA_2} = \frac{C_2 N \sin\alpha}{NA_2 \sin\gamma} = \left(\frac{\sin\gamma}{\sin\alpha}\right)^2 \cdot \frac{\sin\alpha}{\sin\gamma} = \frac{\sin\gamma}{\sin\alpha} \qquad (3)$$

munosabatni hosil qilamiz.

(3) munosabatdan $\Delta\, AIM$ va $\Delta\, CIM$ lar uchun sinuslar teoremasidan foydalanib, $AM = \frac{MI\cdot\sin\alpha}{\sin AIM} = \frac{MI\cdot\sin\alpha}{\sin NIA_2} = \frac{MI\cdot\sin\gamma}{\sin C_2IN} = \frac{MI\cdot\sin\gamma}{\sin MIC} = MC$ ni hosil qilamiz. Shunday qilib, KI to‘g‘ri chiziq AC tomonning o‘rtasidan o‘tishi isbotlandi.

5-masala. $P(x) = x^4 + ax^3 + bx^2 + cx + d$ ko‘phad $(1) = 10$, $P(2) = 20$, $P(3) = 30$ shartlarni qanoatlantiradi. $P(12) + P(-8)$ ning qiymatini hisoblang.

Yechim: $Q(x) = P(x) - 10x$ ko‘phad uchun $Q(1) = Q(2) = Q(3) = 0$

Tengliklar bajarilganligi bois

$Q(x) = P(x) - 10x = (x-1)(x-2)(x-3)(x-A)$ bo‘ladi, bu yerda A-haqiqiy son. Bundan $P(x) = (x-1)(x-2)(x-3)(x-A) + 10x$.

Demak, $P(12) + P(-8) = 11 \cdot 10 \cdot 9 \cdot (12 - A) + 120 + (-9)\cdot(-10)\cdot(-11)\cdot(-8 - A) - 80 = 19840$.

Javob:19840

Xulosa

Olimpiada masalalari o‘quvchilarning analitik tafakkuri, mantiqiy fikrlashi va ijodiy qobiliyatlarini rivojlantiradi.

Ularni muntazam yechish yuqori natijalarga erishishga xizmat qiladi.

FOYDALANILGAN ADABIYOTLAR:

1. Sh.A. Alimov va boshqalar. Algebra.
2. Matematika fanidan olimpiada masalalari to‘plamlari.
3. G. Polya. How to Solve It.
4.http://www.math-on-line.com – Qiziqarli matematika masalalari.
5. http://www.ams.org/mathweb/ - Math on the Web-Internetda matematika.