



AYRIM TRIGONOMETRIK MASALALARNI GEOMETRIK USULDA YECHISH

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Annotatsiya: Ushbu ilmiy maqolada ba'zi trigonometrik ayniatlarni uchburchakning xossaligidan foydalanib hisoblash juda qulay bo'lishi ko'rsatib o'tilgan.

Kalit so'zlar: trigonometriya, geometriya, birlik aylana, sinus, kosinus, trigonometrik tenglama, metodika.

KIRISH

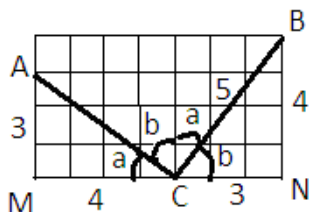
Trigonometrik masalalarni geometrik usulda yechish o'quvchilarning mantiqiy va fazoviy tafakkurini rivojlantiradi.

Asosiy qism

Birlik aylana, to'g'ri burchakli uchburchak hamda sinus va kosinus teoremlari trigonometrik masalalarni geometrik usulda yechishning asosini tashkil etadi. $\sin^2 x + \cos^2 x = 1$ ayniyati birlik aylana orqali tabiiy izohlanadi.

1-masala. Hisoblang: $\arctg \frac{3}{4} + \arccos \frac{3}{5}$

Yechimi. AMC va BNC to'g'ri burchakli uchburchaklarni quyidagicha chizib olamiz (1-rasm).



1-rasm

a va b to'g'ri burchakli uchburchakning o'tkir burchaklari bo'lsin.

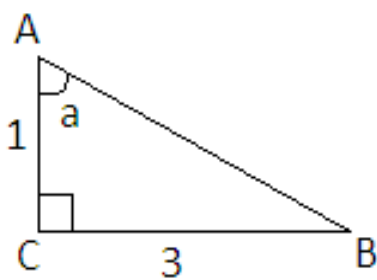
$\operatorname{tga} = \frac{3}{4} \Rightarrow a = \arctg \frac{3}{4}$, $\cos b = \frac{3}{5} \Rightarrow b = \arccos \frac{3}{5}$ ekanligidan, hamda chizmadan ko'rinib turibdiki $a+b=90^\circ$ bo'ladi.

Javob: 90°

2-masala Agar $\operatorname{tga} = 3$ bo'lsa, $\frac{3 \sin a}{5 \sin^3 a + 10 \cos^3 a}$ ifodaning qiymatini toping.

Yechimi. Quyidagi to'g'ri burchakli uchburchakda $\operatorname{tg} \alpha = 3$ ekanligidan,

$\sin \alpha = \frac{3}{\sqrt{10}}$ va $\cos \alpha = \frac{1}{\sqrt{10}}$ natijalarni olamiz.



2-rasm

U holda,
$$\frac{3 \sin a}{5 \sin^3 a + 10 \cos^3 a} = \frac{3 \cdot \frac{3}{\sqrt{10}}}{5 \cdot \left(\frac{3}{\sqrt{10}}\right)^3 + 10 \cdot \left(\frac{1}{\sqrt{10}}\right)^3} = \frac{9}{\frac{27}{2} + 1} = \frac{18}{29} \text{ bo'ladi.}$$

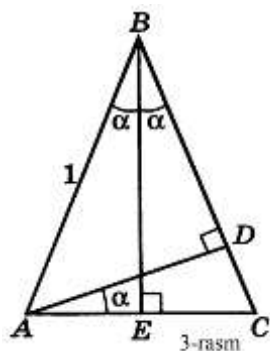
Javob: $\frac{18}{29}$

3-masala. $\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$ formulani keltirib chiqaring.

Yechimi.

Uchidagi burchagi 2α va yon tomoni 1 ga teng bo'lgan ABC teng yonli uchburchakni qaraymiz.

Bu uchburchakning AD va BE balandliklarini o'tkazamiz (3-rasm).



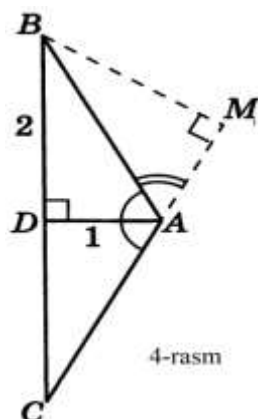
Chizmadan ko'rinib turibdiki, $AD = \sin 2\alpha$, $AE = EC = \sin \alpha$. Bundan tashqari ABE va CAD uchburchaklarning o'xshash ekanligidan, $\frac{AB}{AC} = \frac{BE}{AD}$ bo'ladi.

Bu tengliklardan $\frac{1}{2 \sin \alpha} = \frac{\cos \alpha}{\sin 2\alpha}$ tenglikni hosil qilamiz.

Bundan esa $\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$ formula kelib chiqadi.

4-masala. Hisoblang: $\cos(2\arctg 2)$.

Yechimi. Quyidagi uchburchakdan foydalanamiz(4-rasm):



BAC - teng yonli uchburchakdan $\angle BAC = 2\arctg 2$. Bu burchak o'tmas burchak ekanligidan, $\arctg 2 > \arctg 1 = \frac{\pi}{4}$ bo'ladi.

Demak, $\cos(2\arctg 2) = -\cos \angle BAM$. Bundan esa $\cos \angle BAM = \frac{AM}{AB}$ bo'ladi. Pifagor teoremasiga ko'ra $\triangle BAM$ uchburchakdan $AM = \sqrt{AB^2 - BM^2}$,

$\triangle ABD$ uchburchakdan esa $AB = \sqrt{5}$ natijalarni olamiz.

BM kesma uzunligini ABC uchburchakning balandligi sifatida topish mumkin, ya'ni $BM = \frac{2S_{\triangle ABC}}{AC}$. Demak, $BM = \frac{4}{\sqrt{5}}$. Endi AM kesma uzunligini aniqlaymiz,

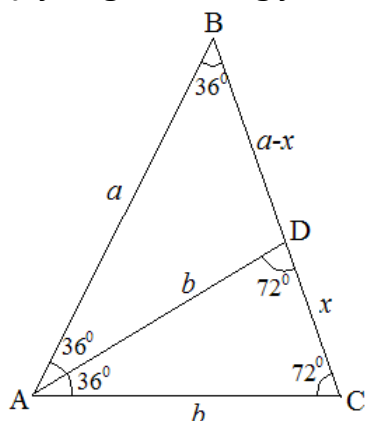
$$\cos(2\arctg 2) = -\frac{3}{5}.$$

Javob: $\frac{3}{5}$

5-masala. $\sin 18^\circ$ ning qiymatini toping.

Yechimi.

Quyidagi ABC teng yonli uchburchakni qaraymiz(5-rasm):



5-rasm

Bu uchburchakning AD -bissektrisasini o'tkazsak, ikkita o'xshash uchburchaklar hosil bo'ladi (ABC va CAD). Bissektrisa xossasidan $x = \frac{ab}{a+b}$ ekanligi ma'lum.



Uchburchaklarning o'xshashlik xossasidan $\frac{a}{a+b} = \frac{b}{a}$ tenglikni hosil qilamiz. Bundan a noma'lumli kvadrat tenglama hosil bo'ladi: $a^2 - ab - b^2 = 0$. Bu tenglamani yechib $a = \frac{\sqrt{5}+1}{2}b$ tenglikni topamiz. ABC uchburchakka sinuslar teoremasini qo'llasak, $\frac{a}{\sin 72^\circ} = \frac{b}{\sin 36^\circ}$ bo'ladi. U holda, $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$ bo'ladi. Bundan esa, $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ ekanligini aniqlaymiz.

$$\text{Javob: } \sin 18^\circ = \frac{\sqrt{5}-1}{4}.$$

Xulosa

Geometrik yondashuv trigonometrik masalalarni sodda va ko'rgazmali yechishga imkon beradi.

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