APPLICATION OF DERIVATIVES IN SOLVING ECONOMIC PROBLEMS

Jalolxon Nuritdinov
Lecturer of Kokand State University
E'zozaxon Abdullayeva
Student of Kokand State University

The use of mathematical methods in economics is becoming increasingly important in science and finance. In particular, the derivative, one of the fundamental concepts of mathematical analysis, is widely applied to determine the rate of change of economic processes, find optimal results, and assess economic efficiency. The derivative serves as a key tool in economic modelling and minimizing losses.

In the textbook "Fundamental Methods of Mathematical Economics" (2005) written by Chiang A.C., and Wainwright K., the economic applications of derivatives are discussed in detail [1]. This article analyzes the practical significance of derivatives in solving economic problems based on that scientific source.

Definition. Let$f(x)$be a real-valued function. The derivative of f at a point x is defined as:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h},$$

if the limit exists. This limit measures how fast the function changes when x changes [2-6].

Other words, A derivative tells us the rate of change of a function. It answers the question: "If I change x a little bit, how much does (x) change?" In simple words: Positive derivative → the function is increasing. Negative derivative → the function is decreasing. Zero derivative → the function is flat (maximum/minimum point).

Another intuitive idea: The derivative is the slope of the tangent line to the graph of (x) at a specific point.

The mathematical derivative is an essential analytical tool for identifying relationships between variables in economics. Using derivatives, it is possible to determine how the outcome of an economic function changes when its argument changes. For example, in the production function $Y = f(L,K)$, where L is labor and K is capital, the derivatives $Y'(L)$ and $Y'(K)$ represent the marginal productivity of labor and capital, respectively. These indicators are important for determining the efficiency of resource use in the production process. If the profit function is $TR - TC$ (where TR is total revenue and TC is total cost), the derivative helps determine the maximum profit. The condition for maximum profit is expressed through the first-order derivative, which identifies the optimal production level. This approach forms the basis of economic optimization theory. Among the economic applications of derivatives, determining the elasticity of demand and supply plays a significant role. If the demand function is expressed as $Q = f(P)$, the price elasticity is found using the formula:

$$E = (dQ/dP) \times (P/Q).$$

This formula measures how changes in price affect demand. Positive elasticity indicates that supply increases as price rises, while negative elasticity shows that demand decreases when the price increases. The elasticity coefficient is widely used in economic policy-making, such as price regulation, taxation, and market equilibrium analysis. Derivatives are also used to determine marginal cost (MC) and marginal revenue (MR). For a total cost function $C(Q)$, the derivative $C'(Q) = dC/dQ$ shows the rate of cost increase. For a total revenue function $R(Q)$, the condition $MR = MC$ must hold for optimal production. This equality is one of the key conditions for determining economic equilibrium. The practical use of derivatives appears not only in microeconomics but also in macroeconomics. Economic growth models, investment and consumption functions, inflation and employment indicators are analyzed using derivatives and differential equations. This allows studying the dynamics of economic systems over time [7,8].

According to “Mathematics in Economics” derivatives help determine tax rates, adjust interest rates, and evaluate capital accumulation. In profit maximization problems with multiple variables, systems of partial derivatives are used. These derivatives help determine a firm's optimal resource allocation strategy.

Derivatives also play an important role in finance. They are used to assess stock value and the rate of change in interest rates. The Black–Scholes model is a well-known example, where differential equations and derivatives are used to determine the price of financial derivatives. This enables risk management and increases investment efficiency. Derivatives are equally important in forecasting economic processes. Trend analysis, production growth rate, inflation rate, employment levels, and GDP dynamics are evaluated using derivatives. This helps form economic policies based on scientific analysis.

Example of Derivative in Economics. Suppose a company’s cost to produce
x units is:

$$C(x) = 200 + 5x + 0.02x^2$$

The marginal cost is the derivative:

$C'(x) = 5 + 0.04x.$

If the company is currently producing $x = 50$ units, then:

$$C'(50) = 5 + 0.04(50) = 7.$$

The cost of producing one more unit (from 50 to 51 units) is approximately $7. This is why derivatives are powerful in economics—they help firms analyze marginal (extra) changes, such as: marginal cost, marginal revenue, marginal profit, elasticity

The derivative is one of the most important mathematical tools in economic analysis, determining the rate and direction of changes in economic processes. It is used to analyze production, profit, cost, demand, supply, and other economic functions. Derivatives are widely applied not only in theoretical but also in practical economic problem-solving.

REFERENCES:

1. Chiang, A. C., & Wainwright, K. (2005). Fundamental Methods of Mathematical Economics (4th ed.). McGraw-Hill Education.

2. Anton, H., Bivens, I., & Davis, S. (2016). Calculus: Early Transcendentals (11th ed.). Wiley.

3. Stewart, J. (2016). Calculus: Early Transcendentals (8th ed.). Cengage Learning.

4. Tursunboy o‘g‘li, N. J., & Axadjon o‘g‘li, A. A. (2023). O ‘zbekistonning jahon savdo tashkilotiga a’zo bo ‘lish uchun uzoq yo ‘li va xitoy tajribasi. Qo ‘qon universiteti xabarnomasi, 1, 43-47.

5. Nuritdinov, J. T. (2023). YARIM TEKISLIKLAR MINKOVSKIY AYIRMASI. QO ‘QON UNIVERSITETI XABARNOMASI, 38-40.

6. Mamatov, M., Nuritdinov, J., & Esonov, E. (2021). Дифференциальные игры дробного порядка с распределенными параметрами. International Scientific Technical Journal" Problems of Control and Informatics", 66(4), 38-47.

7. Mamatov, M. S., & Nuritdinov, J. T. (2020). About some rules for calculating the Minkovsky sum and difference. Bulletin of the Institute of Mathematics, 3, 49-59.

8. Nuritdinov, J., Khaydarov, I., Turdaliyev, S., & Djuraev, I. (2025, October). Application of Minkowski difference to optimal control tasks. In AIP Conference Proceedings (Vol. 3377, No. 1, p. 030001). AIP Publishing LLC.