$W_2^{(7,6)}(0,1)$ GILBERT FAZOSIDA OPTIMAL KVADRATUR FORMULA XATOLIK FUNKSIONALI NORMASI KO‘RINISHINI TOPISH

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Annotatsiya: *Ushbu maqolada Gilbert fazosida Eyler–Makloren tipidagi optimal kvadratur formula qaralgan. Kvadratur formulaning xatolik funksionali uchun Riss teoremasi hamda ekstremal funksiya usuli asosida norma ko‘rinishi topilgan. Xatolik funksionalining normasi orqali Sard ma’nosidagi optimal kvadratur formulani qurish masalasi o‘rganilgan hamda tegishli nazariy natijalar keltirilgan.*

Kalit so‘zlar: *Gilbert fazosi, optimal kvadratur formula, Eyler–Makloren formulasi, xatolik funksionali, Sard masalasi, Riss teoremasi, ekstremal funksiya, sonli integrallash, kvadratur formula normasi, Sobolev fazosi.*

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Eyler-Makloren kvadratur formulasi diskret yig‘indilarni aniq integral bilan bog‘lash uchun ishlatiladi. Bu formula sonli hisoblashlarda, kvadratur formulalar yaratishda va akustikadan kvant fizikagacha bo‘lgan turli sohalarda qo‘llaniladi. Eyler-Makloren formulasining sonli integrallash, akustik muhit tahlili, optimal kvadratur formulalar va sonli hisoblashlarda qo‘llaniladi.

Eyler-Makloren formulasi quyidagicha yoziladi:

$$\sum_{k=a}^{b} f(k) \approx \int_a^b f(x)dx + \frac{f(b)+f(a)}{2} + \sum_{j=1}^{m} \frac{B_{2j}}{(2j)!}\left(f^{(2j-1)}(b) - f^{(2j-1)}(a)\right) + R_m$$

Bu yerda B_{2j} Bernulli sonlari, R_m esa qoldiq had hisoblanadi.

P.L.Butzer va R.L.Stem (1983) tomonidan taqdim etilgan tadqiqotda Eyler-Makloren formulasi yordamida haqiqiy o‘qda integratsiyani baholash va Whittaker-Shannon diskretlash teoremasi o‘rtasidagi bog‘liqlik o‘rganilgan [1]. Ushbu tadqiqotda quyidagi xulosalar chiqarilgan:

- Kvadratur metodlarida aniq integrallarning taqribiy bahosi olish mumkin.
- Integrallash xatoligi funksiyaning hosilalariga bog‘liq ravishda kamayadi.

M.J.Panza (2008) maqolasida Green funksiyalarini hisoblashda Eyler-Makloren formulasidan foydalanilgan [2]. Bu yondashuv:

- To‘lqin tenglamalari uchun yopiq shakldagi yechimlarni olish imkonini beradi.
- Tovush tarqalishini modellashtirishda ishlatilgan.

L.F.Meyers and A.Sard tomonidan taqdim etilgan tadqiqotda optimal kvadratur formulalarini yaratishda Eyler-Makloren kvadratur formulasining roli o‘rganilgan [3]. Ularning yondashuvi:

- Kvadratur formulalarining aniqligini maksimal darajada oshirish imkonini beradi.
- Olingan natijalar umumlashgan trapetsiya va Simpson formulalariga nisbatan aniqroq hisoblanadi.

El-Ajou (2013) tomonidan funksional qatorlar va Eyler-Makloren formulasining umumlashtirilgan shakllari o'rganilgan [4]. Uning ishlarida

- Funksional qatorlar tartibli kvadratur metodlari ishlab chiqilgan.
- Kuchli bog'lanishli differensial tenglamalar yechimlari uchun Eyler-Makloren formulasi asosida yangi usul taqdim etilgan.

Eyler-Makloren kvadratur formulasi turli sohalarda qo'llaniladigan kuchli vosita bo'lib, quyidagi afzalliklarga ega:

- Sonli iintegrallash uchun aniq va yaqinlashuvchi baholar beradi.
- Akustik va fizik muammolarda yopiq shakldagi yechimlar olish imkonini yaratadi.
- Optimal kvadratur formulalarini yaratishda muhim rol o'ynaydi.
- Differensial tenglamalar va sonli usullarda samarali ishlatiladi.

Ushbu maqolada $W_2^{(7,6)}(0,1)$ Gilbert fazosida yangi Eyler-Makloren kvadratur formulasini kengaytirilgan variantni qurilgan va chiziqli funksionallar uchun Riss teoremasi va ekstremal funksiya yordamida xatolik funksionali normasi ko'rinishi topilgan.

1. Masalaning qo'yilishi. Biz quyidagi kvadratur formulani qaraymiz:

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\varphi(h\beta) +$$

$$+\sum_{\beta=0}^{N} C_3[\beta]\varphi'''(h\beta) + \sum_{\beta=0}^{N} C_5[\beta]\varphi^{(5)}(h\beta) + \sum_{\beta=0}^{N} C_6[\beta]\varphi^{(6)}(h\beta) \qquad (1)$$

bunda C_0, C_1, C_3, C_5-ma'lum koeffisentlar. Ya'ni

$$C_0[\beta] = \begin{cases} h/2, \beta = 0 \\ h, \beta = \overline{1, N-1} \\ h/2, \beta = N \end{cases} \qquad C_1[\beta] = \begin{cases} h^2/12, \beta = 0 \\ 0, \beta = \overline{1, N-1} \\ -h^2/12, \beta = N \end{cases}$$

$$C_3[\beta] = \begin{cases} -h^4/720, \beta = 0 \\ 0, \beta = \overline{1, N-1} \\ h^4/720, \beta = N \end{cases} \quad C_5[\beta] = \begin{cases} h^6/30240, \beta = 0 \\ 0, \beta = \overline{1, N-1} \\ -h^6/30240,, \beta = N \end{cases}$$

$C_6[\beta]$-(1) kavdratur formulani hozircha noma'lum koeffisentlari, $h = \dfrac{1}{N}$,

$N-$natural son.

Bu yerda integral ostidagi φ funksiyalar $W_2^{(7,6)*}(0,1)$ fazoga tegishli. Bunda $W_2^{(7,6)}$ fazo sifatida oltinchi tartibli hosilasi absolyut uzluksiz yettinchi tartibli hosilasi $L_2(0,1)$ fazosiga tegishli funksiyalar sinfini olamiz.Bu $W_2^{(7,6)*}(0,1)$ sinfida skalyar ko'paytma quyidagicha kiritilgan:

$$<\varphi,\psi> = \int_0^1 (\varphi^{(7)}(x) + \varphi^{(6)}(x)) * (\psi^{(7)}(x) + \psi^{(6)}(x))dx \qquad (2)$$

yarim skalyar ko'paytmaga nisbatan Gilbert fazosi bo'ladi. Bu skalyar ko'paytma yordamida quyidagi normani qaraymiz:

$$\left\| \varphi \middle| W_2^{(7,6)} \right\| = \left(\int_0^1 (\varphi^{(7)}(x) + \varphi^{(6)}(x))dx \right)^{1/2} \qquad (3)$$

(1) kvadratur formulaning xatoligi deb

$$\int_0^1 \varphi(x)dx - \left(\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta) + \sum_{\beta=0}^{N} C_3[\beta]\varphi'''(h\beta) + \right.$$
$$\left. + \sum_{\beta=0}^{N} C_5[\beta]\varphi^{(5)}(h\beta) + \sum_{\beta=0}^{N} C_6[\beta]\varphi^{(6)}(h\beta) \right) \qquad (4)$$

ayirmaga aytiladi. Bu ayirmaga $W_2^{(7,6)}(0,1)$ fazosida aniqlangan quyidagi funksional mos keladi:

$$\ell(x) = \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0[\beta]\delta(x-h\beta) - \sum_{\beta=0}^{N} C_1[\beta]\delta'(x-h\beta) -$$
$$- \sum_{\beta=0}^{N} C_3[\beta]\delta^{(3)}(x-h\beta) - \sum_{\beta=0}^{N} C_5[\beta]\delta^{(5)}(x-h\beta) - \sum_{\beta=0}^{N} C_6[\beta]\delta^{(6)}(x-h\beta) \qquad (5)$$

bunda $\varepsilon_{[0,1]}(x)$ - [0,1] kesmaning xarakteristik funksiyasi, $\delta(x)$ - Dirakning delta-funksiyasi, ya'ni $\delta(x)$-ham funksional bo'lib, uning biror silliq funksiya ta'siri, ya'ni qiymati quyidagicha aniqlanadi:

$$\left(\delta(x), \varphi(x)\right) = \varphi(0), \quad \left(\delta(x-a), \varphi(x)\right) = \varphi(a), \quad \left(\delta^{(\alpha)}(x-a), \varphi(x)\right) = (-1)^{\alpha} \cdot \varphi^{(\alpha)}(a)$$

Isbot: Bu yerda $\ell(x)$ funksionalning $\varphi(x)$ funksiyadagi qiymati quyidagicha aniqlanadi:

$$\left(\ell(x), \varphi(x)\right) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx. \qquad (6)$$

(6)-tenglikka asosan (5)-ni hisobga olib, haqiqattan ham (4)-ayirma $\ell(x)$ xatolik funksionalning $\varphi(x)$ dagi qiymati ekanligiga ishonch hosil qilamiz, ya'ni

$$(\ell, \varphi) = \int_{-\infty}^{\infty} \ell(x) \cdot \varphi(x)dx = \left(\int_{-\infty}^{\infty} \varepsilon_{[0,1]}(x) - \sum_{\beta=0}^{N} C_0[\beta]\delta(x-h\beta) + \sum_{\beta=0}^{N} C_1[\beta]\delta'(x-h\beta) + \right.$$
$$\left. + \sum_{\beta=0}^{N} C_3[\beta]\delta^{(3)}(x-h\beta) + \sum_{\beta=0}^{N} C_5[\beta]\delta^{(5)}(x-h\beta) + \sum_{\beta=0}^{N} C_6[\beta]\delta^{(6)}(x-h\beta) \right) \cdot \varphi(x)dx =$$
$$= \int_0^1 \varphi(x)dx - \left(\sum_{\beta=0}^{N} C_0[\beta] \int_{-\infty}^{\infty} \delta(x-h\beta) \cdot \varphi(x)dx + \sum_{\beta=0}^{N} C_1[\beta] \int_{-\infty}^{\infty} \delta'(x-h\beta) \cdot \varphi(x)dx + \right.$$
$$+ \sum_{\beta=0}^{N} C_3[\beta] \int_{-\infty}^{\infty} \delta^{(3)}(x-h\beta) \cdot \varphi(x)dx + \sum_{\beta=0}^{N} C_5[\beta]\delta^{(5)}(x-h\beta) \cdot \varphi(x)dx +$$
$$\left. + \sum_{\beta=0}^{N} C_6[\beta]\delta^{(6)}(x-h\beta) \cdot \varphi(x)dx) \right) =$$

$$=\int_0^1 \varphi(x)dx-\left(\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)-\sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta)+\right.$$

$$\left.+\sum_{\beta=0}^{N} C_3[\beta]\varphi'''(h\beta)+\sum_{\beta=0}^{N} C_5[\beta]\varphi^{(5)}(h\beta)+\sum_{\beta=0}^{N} C_6[\beta]\varphi^{(6)}(h\beta)\right)$$

Demak, yuqoridagi natijadan ko‘rinadiki biz (4) xatolikni quyidagicha yozib olishimiz mumkin

$$(\ell,\varphi)=\int_0^1 \varphi(x)dx-\left(\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)++\sum_{\beta=0}^{N} C_1[\beta]\varphi'(h\beta)+\right.$$

$$\left.+\sum_{\beta=0}^{N} C_3[\beta]\varphi'''(h\beta)+\sum_{\beta=0}^{N} C_5[\beta]\varphi^{(5)}(h\beta)+\sum_{\beta=0}^{N} C_6[\beta]\varphi^{(6)}(h\beta)\right).$$

(1) kvadratur formulaning ℓ xatolik funksionali $W_2^{(7,6)*}(0,1)$ qo’shma fazoga tegishli chiziqli funksionaldir. Shu bilan birga, (5) xatolik funksionali $W_2^{(7,6)}(0,1)$ fazoda aniqlanganligi uchun quyidagi tengliklarni qanoatlantirishi shartdir:

$$(\ell,1)=1-\sum_{\beta=0}^{N} C_0[\beta]=0, \qquad (7)$$

$$(\ell,x)=\frac{1}{2}-\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)=0, \qquad (8)$$

$$(\ell,x^2)=\frac{1}{3}-\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)^2-2\sum_{\beta=0}^{N} C_1[\beta](h\beta)=0, \qquad (9)$$

$$(\ell,x^3)=\frac{1}{4}-\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)^3-3\sum_{\beta=0}^{N} C_1[\beta](h\beta)^2-6\sum_{\beta=0}^{N} C_3[\beta]=0, \qquad (10)$$

$$(\ell,x^4)=\frac{1}{5}-\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)^4-4\sum_{\beta=0}^{N} C_1[\beta](h\beta)^3-24\sum_{\beta=0}^{N} C_3[\beta](h\beta)=0, \qquad (11)$$

$$(\ell,x^5)=\frac{1}{6}-\sum_{\beta=0}^{N} C_0[\beta]\varphi(h\beta)^5-5\sum_{\beta=0}^{N} C_1[\beta](h\beta)^4-$$

$$-60\sum_{\beta=0}^{N} C_3[\beta](h\beta)^2+120C_5[\beta]=0 \qquad (12)$$

$$(\ell,e^{-x})=1-e^{-1}-\sum_{\beta=0}^{N} C_0[\beta]\cdot e^{-(h\beta)}+\sum_{\beta=0}^{N} C_1[\beta]\cdot e^{-(h\beta)}+$$

$$+\sum_{\beta=0}^{N} C_3[\beta]\cdot e^{-(h\beta)}+\sum_{\beta=0}^{N} C_5[\beta]\cdot e^{-(h\beta)}+\sum_{\beta=0}^{N} C_6[\beta]\cdot e^{-(h\beta)} \qquad (13)$$

Yuqoridagi (7)-(12) tengliklardan (1) kvadratur formulaning beshinchi tartibli ko‘phadga aniqligini bildiradi. Biz $C_6[\beta]$ koeffisentlarni (7)-(12) shartlarni bajariladigan qilib tanlab oldik. Hozirgi noma’lum bo‘lgan $C_6[\beta],\ \beta=0,1,..,N$ koeffisentlar (13) tenglikni qanoatlantiradigan qilib topishimiz kerak. Demak biz $C_6\left[\beta\right]$ koeffisentlar uchun faqatgina (13)-shartga ega bo‘lamiz.

(1) kvadratur formulaning (6) xatoligi $W_2^{(7,6)*}(0,1)$ fazosidan chiziqli funksional bo‘ladi, bunda $W_2^{(7,6)*}$ -bu $W_2^{(7,6)}$ fazosiga qo‘shma fazo. U holda Koshi-Shvars tengsizligidan (6) xatolikning absolyut qiymati yuqoridan quyidagicha baholanadi:

$$\left|(\ell,\varphi)\right| \le \left\|\varphi \,|\, W_2^{(7,6)}(0,1)\right\| \cdot \left\|\ell \,|\, W_2^{(7,6)*}(0,1)\right\|.$$

Bu yerdan biz (1) kvadratur formulaning (6) xatoligi $\ell(x)$ xatolik funksionalining

$$\left\|\ell \,|\, W_2^{(7,6)*}(0,1)\right\| = \sup_{\varphi,\|\varphi\|\neq 0} \frac{\left|(\ell,\varphi)\right|}{\left\|\varphi \,|\, W_2^{(7,6)}(0,1)\right\|}$$

normasi orqali baholanishini xulosa qilamiz. Bu yerda $\ell(x)$ xatolik funksionalining normasi $C_6[\beta],\ \beta=\overline{0,N}$ koeffisentlarga bog‘liqligini ko‘rish qiyin emas. Tabiiyki, butun bir funksiyalar fazosi $W_2^{(7,6)}(0,1)$ elementlarida (1) kvadratur formula xatoligini yuqori chegarasining eng kichik qiymatini topish muhim hisoblanadi. Xatolik funksionali normasi $C_6[\beta]$ koeffisentlarga bo‘yicha minimumini topish masalasi Sard masalasi bo‘ladi.

Bu masalani yechish natijasida olingan kvadratur formula Sard manosida optimal kvadratur formula deyiladi.

Ushbu ishning asosiy maqsadi $W_2^{(7,6)}(0,1)$ fazosida (1) kvadratur formula uchun Sard masalasi yechishdan iboratdir, ya’ni

$$\left\|\overset{\circ}{\ell}\right\| = \inf_{C_6[\beta]} \|\ell\| \qquad (14)$$

tenglikni qanoatlantiruvchi $C_6[\beta]$ koeffisentlarni topishdan iboratdir.

Shunday qilib, $W_2^{(6,5)}(0,1)$ fazosida (1) ko‘rinishidagi Sard manosida optimal kvadratur formulani qurish uchun quyidagi ikkita masalani ketma-ket yechishimiz kerak.

1 – masala. (1) kvadratur formulaning (5) xatolik funksionali normasi hisoblash.

2 – masala. (14) tenglikni qanoatlantiruvchi $C_6[\beta]$ koeffisentlar topish.

Biz quyida birinchi masalani yechish bilan shug‘ullanamiz.

2. Xatolik funksionali normasini hisoblash.

$\ell(x)$ xatolik funksionali normasi hisoblash uchun biz quyidagi tenglikni qanoatlantiruvchi ψ_ℓ ekstremal funksiyadan foydalanamiz

$$(\ell,\psi_\ell) = \left\|\ell \,|\, W_2^{(7,6)*}(0,1)\right\| \cdot \left\|\psi_\ell \,|\, W_2^{(7,6)}(0,1)\right\|$$

Shuni ham ta’kidlash kerakki, $W_2^{(7,6)}(0,1)$ Gilbert fazosida aniqlangan har qanday ℓ chiziqli funksional uchun [1] ishda ψ_ℓ ekstremal funksiya topilgan va ekstremal funksiya quyidagi chegaraviy masalaning yechimi ekanligi ko‘rsatilgan:

$$\psi_\ell^{(2m)}(x) - \psi_\ell^{(2m-2)}(x) = (-1)^m \ell(x) \qquad (15)$$

$$\left(\psi_\ell^{(m+s)}(x)-\psi_\ell^{(m+s-2)}(x)\right)\Big|_{x=0}^{x=1}=0, \qquad s=0,1,...,m-1, \tag{16}$$

$$\left(\psi_\ell^{(m)}(x)-\psi_\ell^{(m-1)}(x)\right)\Big|_{x=0}^{x=1}=0 \tag{17}$$

hamda ψ_ℓ ekstremal funksiya uchun quyidagi teorema isbotlangan:

2.1-teorema [17]. (15) – (17) –chegaraviy masalaning yechimi ℓ xatolik funksionali uchun ψ_ℓ ekstremal funksiya bo‘ladi va u quyidagi ko‘rinishda bo‘ladi

$$\psi_\ell(x)=(-1)^m\ell(x)*G_m(x)+P_{m-2}(x)+de^{-x},$$

bunda

$$G_m(x)=\frac{\mathrm{sign}x}{2}\left(\frac{e^x-e^{-x}}{2}-\sum_{k=1}^{m-1}\frac{x^{2k-1}}{(2k-1)!}\right),$$

$P_{m-2}(x)$ - ixtiyoriy $m-2$ darajali ko‘phad, d ixtiyoriy haqiqiy son.

Bundan tashqari quyidagi tenglik o‘rinli bo‘lishi ko‘rsatilgan. Bu tengliklar chiziqli funsionallar uchun Riss teoremasidan kelib chiqqan

$$\left\|\ell\,|\,W_2^{(m,m-1)*}\right\|=\left\|\psi_\ell\,|\,W_2^{(m,m-1)*}\right\| \text{ va } (\ell,\psi_\ell)=\left\|\ell\,|\,W_2^{(m,m-1)*}\right\|^2. \tag{18}$$

Endi biz qarayotgan $W_2^{(7,6)}(0,1)$ fazosida ℓ xatolik funksionali uchun ekstremal funksiyani yuqoridagi teoremadan $m=7$ holda olamiz.

Shunday qilib, 2.1-teoremadan $m=7$ holda $W_2^{(7,6)}(0,1)$ fazosida aniqlangan ℓ xatolik funksionali uchun ψ_ℓ ekstremal funksiyani olamiz va u quyidagicha bo‘ladi

$$\psi_\ell(x)=-\ell(x)*G_7(x)+P_5(x)+de^{-x}, \tag{19}$$

bunda $P_5(x)$ beshinchi darajali ko‘phad va

$$G_6(x)=\frac{\mathrm{sgn}\,x}{2}\cdot\left(\frac{e^x-e^{-x}}{2}-\frac{x^{11}}{11!}-\frac{x^9}{9!}-\frac{x^7}{7!}-\frac{x^5}{5!}-\frac{x^3}{3!}-x\right) \tag{20}$$

Birinchi masalaning yechimi uchun quyidagi o‘rinli.

2.2-teorema. (1) kvadratur formula (5) xatolik funksionali normasi uchun quyidagi ifoda o‘rinli

$$\|\ell\|^2=(\ell,\psi_\ell)=\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_6[\beta]C_6[\gamma]G_7^{(12)}(h\beta-h\gamma)+$$

$$+2\sum_{\beta=0}^{N}C_6[\beta]\left(\int_0^1 G_7^{(6)}(x-h\beta)dx-\sum_{\gamma=0}^{N}C_0[\gamma]G_7^{(6)}(h\beta-h\gamma)-\sum_{\gamma=0}^{N}C_1[\gamma]G_7^{(7)}(h\beta-h\gamma)-\right.$$

$$\left.-\sum_{\gamma=0}^{N}C_3[\gamma]G_7^{(9)}(h\beta-h\gamma)-\sum_{\gamma=0}^{N}C_5[\gamma]G_7^{(11)}(h\beta-h\gamma)\right)-\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_0[\beta]C_0[\gamma]G_7(h\beta-h\gamma)+$$

$$+2\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_0[\beta$$

$$+2\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_0[\beta]C_5[\gamma]G_7^{(5)}(h\beta-h\gamma)+\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_1[\beta]C_1[\gamma]G_7^{"}(h\beta-h\gamma)+ \tag{21}$$

$$+2\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_1[\beta]C_3[\gamma]G_7^{(4)}(h\beta-h\gamma)+2\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_1[\beta]C_5[\gamma]G_7^{(6)}(h\beta-h\gamma)+$$

$$+\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_3[\beta]C_3[\gamma]G_7^{(6)}(h\beta-h\gamma)+\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_3[\beta]C_5[\gamma]G_7^{(8)}(h\beta-h\gamma)+$$

$$+\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}C_5[\beta]C_5[\gamma]G_7^{(10)}(x-h\beta)+2\sum_{\beta=0}^{N}C_0[\beta]\int_0^1 G_7(x-h\beta)dx-$$

$$-2\sum_{\beta=0}^{N}C_1[\beta]\int_0^1 G_7'(x-h\beta)dx-2\sum_{\beta=0}^{N}C_3[\beta]\int_0^1 G_7'''(x-h\beta)dx-2\sum_{\beta=0}^{N}C_5[\beta]\int_0^1 G_7^{(5)}(x-h\beta)dx-$$

$$-\int_0^1\int_0^1 G_7(x-y)dxdy.$$

Bu yerda $G_7(x)$ funsiyaning hosilalarini hisoblashda quyidagi formuladan foydalanamiz

$$G_m^{(t)}(x)=\frac{\operatorname{sgn}x}{2}\begin{cases}\frac{e^x-e^{-x}}{2}-\frac{x^{2m-3-t}}{(2m-3-t)!}-\ldots-\frac{x^3}{3!}-x, agar\quad t-\quad juft\quad son\quad bo'lsa,\\ \frac{e^x+e^{-x}}{2}-\frac{x^{2m-3-t}}{(2m-3-t)!}-\ldots-\frac{x^2}{2!}-1, agar\quad t-\quad juft\quad son\quad bo'lmasa.\end{cases}$$

(22)

Isbot. (21) ifodani (18) tenglik asosida $m=7$ bo‘lganda, (5) ва (19) tengliklarni inobatga olib hisoblaymiz.

Demak,

$$\|\ell\|^2=(\ell,\psi_\ell)=\left(\ell,-\ell(x)\cdot G_7(x)+P_5+de^{-x}\right)=(\ell,-\ell(x)\cdot G_7)+P_5+d\cdot e^{-x}$$

Bu yerdan (7)- (11) tengliklarni hisobga olib quyidagiga kelamiz

$$\|\ell\|^2=(l,\psi_7)=-(\ell,\ell(x)\cdot G_7(x)) \qquad (23)$$

Bundan ko‘rinadiki, $\|\ell\|^2$ ni hisoblash uchun dastlab $\ell*G_7$ ni hisoblaymiz. Quyidagiga egamiz:

$$(\ell\cdot G_7)(x)=\int_{-\infty}^{\infty}\ell(y)\cdot G_7(x-y)dy=$$

$$=\int_0^1 G_7(y-x)dy-\sum_{\beta=0}^{N}C_0[\beta]\cdot G_7(h\beta-x)-\sum_{\beta=0}^{N}C_1[\beta]\cdot G_1'(h\beta-x)-$$

$$-\sum_{\beta=0}^{N}C_3[\beta]\cdot G_7^{(3)}(h\beta-x)-\sum_{\beta=0}^{N}C_3[\beta]\cdot G_7^{(5)}(h\beta-x)-\sum_{\beta=0}^{N}C_6[\beta]\cdot G_7^{(6)}(h\beta-x)=$$

$$=\int_0^1 G_7(x-y)dy-\sum_{\beta=0}^{N}C_0[\beta]\cdot G_7(x-h\beta)+\sum_{\beta=0}^{N}C_1[\beta]\cdot G_7'(x-h\beta)+$$

$$+\sum_{\beta=0}^{N}C_3[\beta]\cdot G_7^{(3)}(x-h\beta)+\sum_{\beta=0}^{N}C_3[\beta]\cdot G_7^{(5)}(x-h\beta)-\sum_{\beta=0}^{N}C_6[\beta]\cdot G_7^{(6)}(x-h\beta)$$

endi oxirgi tenglikni inobatga olib (19) ifodadan $\|\ell\|^2$ uchun quyidagini olamiz:

$$\|\ell\|^2=(\ell,\psi_\ell)=-(\ell,\ell*G_7)=-\int_{-\infty}^{\infty}\ell(x)\cdot(\ell\cdot G_7)(x)dx$$

Yuqoridagi ifoda ustida ma'lum soddalashlarni amalga oshirib, (21) tenglikni hosil qilamiz. Teorema isbotlandi.

Birinchi masala yechildi.

FOYDALANILGAN ADABIYOTLAR:

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